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# On two generalized Laplace transforms.

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# On two generalized Laplace transforms.

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**Summary.** - *In this paper some convergence theorems are established for the transforms*

$$f(s)=\int_0^\infty e^{-1/2\, st}(st)^{m-1/2}W_{k,\,m}(st)\,d\alpha(t)$$

*and*

$$f_1(s)=\int_0^\infty e^{-1/2\, st}(st)^{-k-1/2}W_{k+1/2,m}(st)\,d\alpha(t),$$

*which are generalizations of the* LAPLACE *transform*

$$f_0(s)=\int_0^\infty e^{-st}d\alpha(t).$$


## 1. - Introduction.

The LAPLACE-STIELTJES transform in the classical form is

$$f_0(s)=\int_0^\infty e^{-st}d\alpha(t) \tag{1.1}$$

where $\alpha(t)$ is a function of bounded variation in $0\leqq t\leqq R$ for every positive $R$.

VARMA (1951) has given a generalization of (1.1) in the form

$$f_1(s)=\int_0^\infty e^{-\frac{1}{2}st}(st)^{m-\frac{1}{2}}\,W_{k,\,m}(st)\,d\alpha(t) \tag{1.2}$$

which reduces to (1.1) when $k+m=\dfrac{1}{2}$. It can also be represented in the form

$$f_1(s)=\int_0^\infty e^{-st}(st)^{\,m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ st\right)d\alpha(t) \tag{1.3}$$

where $\Psi$ denotes TRICOMI's confluent hypergeometric function given by the relation

$$W_{k,\,m}(z)=e^{-\frac{1}{2}z}z^{m+\frac{1}{2}}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ z\right). \tag{1.4}$$

MEIJER (1941) has given another generalization of (1.1) in the form

$$f(s)=\int_0^\infty e^{-\frac{1}{2}st}(st)^{-k-\frac{1}{2}}W_{k+\frac{1}{2},\,m}(st)d\alpha(t).$$

On replacing $k+\frac{1}{2}$ by $k$, we get

$$(1.5)\qquad f_2(s)=\int_0^\infty e^{-\frac{1}{2}st}(st)^{-k}W_{k,\,m}(st)d\alpha(t)$$

$$=\int_0^\infty e^{-st}(st)^{-k+m+\frac{1}{2}}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ st\right)d\alpha(t).$$

The transform (1.5) reduces to (1.1) when $k\pm m=\frac{1}{2}$.

In this paper we establish some convergence theorems for the transforms (1.3) and (1.5). The corresponding theorems for (1.1) can be deduced as particular cases of these theorems.

## 2. - Convergence theorems.

THEOREM 2.1. - If

$$\underset{0\leq u<\infty}{\text{l.u.b.}}\left|\int_0^u e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)d\alpha(t)\right|=M<\infty$$

then (1.3) converges for every $s(=\sigma+i\tau)$ for which $\sigma>\sigma_0$. $(s_0=\sigma_0+i\tau_0)$, provided that

(i) $\frac{1}{2}-k\pm m\neq 0$ or a negative integer,

or

(ii) $\frac{1}{2}-k-m=0$, $\operatorname{Re} m>0$.

PROOF. - Let us set

$$\beta(t)=\int_0^t e^{-s_0u}(s_0u)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0u\right)d\alpha(u).$$

Then

$$I = \int_0^R e^{-st}(st)^{2m}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ st\right)d\alpha(t)$$

$$= (s/s_0)^{2m}\int_0^R \frac{e^{-st}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ st\right)}{e^{-s_0t}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0t\right)}\,d\beta(t)$$

$$= (s/s_0)^{2m}\left[\beta(t)\,\frac{e^{-st}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ st\right)}{e^{-s_0t}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0t\right)}\right]_0^R -$$

$$- (s/s_0)^{2m}\int_0^R \beta(t)(d/dt)\left[\frac{e^{-st}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ st\right)}{e^{-st}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0t\right)}\right]dt.$$

But we have $\beta(\infty) = f(s_0)$ and

$$(2\ 1)\qquad \left(\frac{s}{s_0}\right)^{2m}\frac{e^{-sR}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ sR\right)}{e^{-s_0R}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0R\right)} \sim \left(\frac{s}{s_0}\right)^{k+m-\frac{1}{2}} e^{-R(s-s_0)} \qquad (R \to \infty)$$

$$= o(1) \qquad (R \to \infty),$$

when $\sigma > \sigma_0$, since (ERDÉLYI, 1953, p. 278)

$$(2.2)\qquad \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ x\right) \sim x^{k-m-\frac{1}{2}} \qquad (x \to \infty).$$

Thus the integrated part vanishes.
Also let us set

$$J = \left(\frac{s}{s_0}\right)^{2m}\int_0^\infty \beta(t)\left(\frac{d}{dt}\right)\left[\frac{e^{-st}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ st\right)}{e^{-s_0t}\Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0t\right)}\right]dt.$$

Then

$$|J| \leqq M \left| \left(\frac{s}{s_0}\right)^{2m} \int_0^\infty \left(\frac{d}{dt}\right) \left[ \frac{e^{-st}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ st\right)}{e^{-s_0 t}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0 t\right)} \right] dt \right|$$

where $M$ is the upper bound of $|\beta(t)|$ in $0 \leqq t < \infty$;

$$\leqq M \left| \left(\frac{s}{s_0}\right)^{2m} \left[ \lim_{t\to\infty} \frac{e^{-st}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ st\right)}{e^{-s_0 t}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0 t\right)} - \right.\right.$$

$$\left.\left. - \lim_{t\to 0} \frac{e^{-st}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ st\right)}{e^{-s_0 t}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0 t\right)} \right] \right|,$$

$$\leqq M \left| \left(\frac{s}{s_0}\right)^{2m} \lim_{t\to 0} \left[ \frac{e^{-st}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ st\right)}{e^{-s_0 t}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0 t\right)} \right] \right|$$

by using (2.1).

Now let us take

$$U \equiv \Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ x\right).$$

Then (Erdelyi, 1953, p. 262) as $x$ tends to zero, and

(A) $\frac{1}{2}-k \pm m \neq 0$ or a negative integer, $2m \neq 0$ or an integer,

$$U \sim \frac{\Gamma(-2m)}{\Gamma\left(\frac{1}{2}-k-m\right)} + \frac{\Gamma(2m)}{\Gamma\left(\frac{1}{2}-k+m\right)} x^{-2m}; \tag{2.3}$$

or

(B) $\frac{1}{2}-k \pm m \neq 0$ or a negative integer, $2m = 0$ or a positive integer

$$(2.4)\qquad U \sim \frac{(-1)^{2m-1}}{(2m)!\ \Gamma\left(\frac{1}{2}-k-m\right)}\left[\log x + \psi\left(\frac{1}{2}-k+m\right) - \right.$$

$$\left. - \psi(2m+1) - \psi(1)\right] + \frac{(2m-1)!}{\Gamma\left(\frac{1}{2}-k+m\right)} x^{-2m};$$

(the last term is to be omitted when $2m=0$); or

(C) $\frac{1}{2}-k\pm m \neq 0$ or a negative integer, $2m = a$ negative integer

$$(2.5)\qquad U \sim \frac{(-1)^{-2m-1}}{(-2m)!\ \Gamma\left(\frac{1}{2}-k+m\right)} x^{-2m}\left[\log x + \psi\left(\frac{1}{2}-k-m\right) - \right.$$

$$\left. - \psi(-2m+1) - \psi(1)\right] + \frac{(-2m-1)!}{\Gamma\left(\frac{1}{2}-k-m\right)}$$

(the last term is to be omitted when $2m=-1$); or

(D) $\frac{1}{2}-k-m=0$, $\operatorname{Re} m > 0$

$$(2.6)\qquad U = x^{-2m}\cdot\left(\psi(z) = \frac{d}{dx}\log\Gamma(x)\right).$$

Using the results (2.3), (2.4), (2.5) and (2.6) we have

$$(2.7)\qquad \lim_{t\to 0}\frac{e^{-st}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ st\right)}{e^{-s_0t}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)} = \left(\frac{s}{s_0}\right)^{-2m} \text{ or } 1.$$

Thus we see that $J$ is absolutely convergent and our theorem is established.

COROLLARY 2.1a. – If the integral (1.3) converges for $s_0=\sigma_0+i\tau_0$, it converges for all $s=\sigma+i\tau$ for which $\sigma>\sigma_0$.

COROLLARY 2.1b. – The region of convergence of (1.3) is a half plane.

THEOREM 2.1 shows that the divergence of (1.3) at $s = s_0$ implies its divergence at all $s$ for which $\sigma < \sigma_0$. Hence three cases may arise:

(*i*) (1.3) converges for every point;

(*ii*) (1.3) converges for no point;

(*iii*) (1.3) converges for $\sigma > \sigma_c$ and diverges for $\sigma < \sigma_c$.

We may define $\sigma_c$ as the abscissa of convergence.

We can prove a similar theorem for the transform (1.5).

THEOREM 2.2 – If the integral (1.5) converges for a point $s = s_0 (= \sigma_0 + i\tau_0)$, then it converges for every $s (= \sigma + i\tau)$ for which $\sigma_0 > \sigma_0$, provided that

(*i*) $\frac{1}{2} - k \pm m \neq 0$ or a negative integer,

or

(*ii*) $\frac{1}{2} - k - m = 0$. $\operatorname{Re} m > 0$.

The proof of this theorem is similar to that of the previous theorem.

We now consider the necessary conditions imposed on $\alpha(t)$ by the convergence of integrals (1.3) and (1.5).

THEOREM 2.3. – If the integrals (1.3) converges for $s = s_0 = \gamma + i\delta$ with $\gamma > 0$, then

$$\alpha(t) = o(e^{\gamma t} t^{-k-m+\frac{1}{2}}) \qquad (t \to \infty). \tag{2.8}$$

Proof: Let us set

$$\beta(t) = \int_1^t e^{-s_0 u} (s_0 u)^{2m} \Psi\left(\frac{1}{2} - k + m,\ 2m + 1;\ s_0 u\right) d\alpha(u) \qquad (1 < t < \infty).$$

Then

$$\alpha(t) - \alpha(1) = \int_1^t d\alpha(t) = \int_1^t \left[e^{-s_0 u} (s_0 u)^{2m} \Psi\left(\frac{1}{2} - k + m,\ 2m + 1;\ s_0 u\right)\right]^{-1} d\beta(u) =$$

$$= \beta(t) \left[e^{-s_0 t} (s_0 t)^{2m} \Psi\left(\frac{1}{2} \quad k + m,\ 2m + 1;\ s_0 t\right)\right]^{-1} -$$

$$- \int_1^t \beta(u) \frac{d}{du} \left[e^{-s_0 u} (s_0 u)^{2m} \Psi\left(\frac{1}{2} - k + m,\ 2m + 1;\ s_0 u\right)\right]^{-1} du.$$

By using the Mean Value Theorem of Integral Calculus, we have

$$[\alpha(t)-\alpha(1)]e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)=$$

$$=\beta(t)-\beta(t_1)\left[1-\frac{e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)}{e^{-s_0}s_0^{\,2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0\right)}\right]\qquad(0<t_1<t)$$

$$\sim\beta(\infty)-\beta(\infty)\qquad(t\to\infty),$$

since

$$e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)\sim e^{-s_0t}(s_0t)^{k+m-\frac{1}{2}}\qquad(t\to\infty)$$

$$=o(1)\qquad(t\to\infty).$$

Hence using (2.2) we have

$$\alpha(t)-\alpha(1)=o(e^{\gamma t}t^{-k-m+\frac{1}{2}})\qquad(t\to\infty),$$

from which the result follows.

Similarly we can prove the following theorem for the integral (1.5).

THEOREM 2.4. - If the integral (1.5) converges for $s=s_0=\gamma+i\delta$ with $\gamma>0$, then

$$\alpha(t)=o(e^{\gamma t})\qquad(t\to\infty).\tag{2.9}$$

The proof of this theorem is similar to that of the Theorem 2.3.

THEOREM 2.5. - If the integral (1.3) converges for $s=s_0=\gamma+i\delta$ with $\gamma<0$, and if $\alpha(\infty)$ exists, then

$$\alpha(t)-\alpha(\infty)=o(e^{\gamma t}t^{-k-m+\frac{1}{2}})\qquad(t\to\infty).\tag{2.10}$$

Further the convergence of (1.3) for $s=s_0=\gamma+i\delta$ with $\gamma<0$ implies the existence of $\alpha(\infty)$, if $(i)\ \frac{1}{2}-k\pm m\neq 0$ or a negative integer, $\mathrm{Re}\,m>0$; or $(ii)\ \frac{1}{2}-k-m=0$, $\mathrm{Re}\,m>0$.

Proof: Let us set

$$\beta(t)=\int_1^t e^{-s_0u}(s_0u)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0u\right)d\alpha(u)\qquad(1<t<\infty).$$

Then

$$-\alpha(t)+\alpha(\infty)=\int_t^\infty d\alpha(u)=$$

$$\int_t^\infty \left[e^{-s_0u}(s_0u)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0u\right)\right]^{-1} d\beta(u)=$$

$$=-\frac{\beta(t)}{e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)}-$$

$$-\int_t^\infty \beta(u)\frac{d}{du}\left[e^{-s_0u}(s_0u)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0u\right)\right]^{-1} du,$$

for $\beta(\infty)$ is finite and

$$\left[e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)\right]^{-1}\sim e^{s_0t}(s_0t)^{\frac{1}{2}-k-m} \qquad (t\to\infty)$$

$$=o(1) \qquad (t\to\infty),$$

since $\operatorname{Re} s_0=\gamma<0$.

Therefore

$$[\alpha(\infty)-\alpha(t)]e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)=$$

$$=-\beta(t)-e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)\times$$

$$\times\int_t^\infty \beta(u)\frac{d}{du}\left[e^{-s_0u}(s_0u)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0u\right)\right]^{-1} du.$$

By Mean Value Theorem of Integral Calculus we have

$$[\alpha(\infty)-\alpha(t)]e^{-s_0t}(s_0t)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0t\right)=$$

$$=-\beta(t)+\beta(t_1) \qquad (t<t_1<\infty),$$

since

$$\left[e^{-s_0u}(s_0u)^{2m}\Psi\left(\frac{1}{2}-k+m,\ 2m+1;\ s_0u\right)\right]^{-1}=o(1) \qquad (u\to\infty).$$

Hence using (2.2) we have

$$\alpha(\infty)-\alpha(t)=o(e^{\gamma t}t^{-k-m+\frac{1}{2}}) \qquad (t\to\infty).$$

Further, we have (ERDÉLYI, 1953, p. 256)

$$(2.11)\quad \Psi(a, c; z) = \frac{\Gamma(1-c)}{\Gamma(a-c+1)}\Phi(a, c; z) + \frac{\Gamma(c-1)}{\Gamma(a)} z^{1-c}\Phi(a-c+1, 2-c; z),$$

$$= o(z^{1-c}) \qquad (\mathrm{Re}\, c > 1) \quad (z \to 0)$$

if neither $a - c + 1$ nor $a$ is zero or a negative integer and $c$ is not an integer.

Since $\gamma$ is negative, then by Corollary 2.1a and the relation (2.11) the integral (1.3) converges for $s = 0$ so that $\alpha(\infty)$ exists.

We can prove a similar theorem for the integral (1.5).

THEOREM 2.6 - If the integral (1.5) converges for $s = s_0 = \gamma + i\delta$ with $\gamma < 0$, and $\alpha(\infty)$ exists, then

$$(2.12)\qquad \alpha(t) - \alpha(\infty) = o(e^{\gamma t}) \qquad (t \to \infty).$$

THEOREM 2.7 - If the integral (1.5) converges for $s = s_0 = \gamma + i\delta$, then, provided that $\frac{1}{2} \quad k \pm m \neq 0$ or a negative integer

$$(2.13a)\qquad \alpha(t) = o(t^{k+m-\frac{1}{2}}) \qquad (t \to 0)$$

if $\mathrm{Re}\, m \geqq 0$ $(m \neq 0)$ and $\mathrm{Re}\left(-m - k + \frac{1}{2}\right) > 0$; or

$$(2.13b)\qquad \alpha(t) = o(t^{k-m-\frac{1}{2}}) \qquad (t \to 0)$$

if $\mathrm{Re}\, m < 0$ and $\mathrm{Re}\left(-k + m + \frac{1}{2}\right) > 0$; or

$$(2.13c)\qquad \alpha(t) = o(t^{k-\frac{1}{2}}/\log t) \qquad (t \to 0)$$

if $m = 0$.

Proof: Let us set

$$\gamma(t) = \int_t^1 e^{-s_0 u}(s_0 u)^{\frac{1}{2}-k+m}\, \Psi\left(\frac{1}{2} - k + m,\ 2m + 1;\ s_0 u\right) d\alpha(u) \quad (t < u < 1).$$

Then

$$\alpha(1) - \alpha(t) = \int_t^1 d\alpha(t) =$$

$$= - \int_t^1 \left[ e^{-s_0 t} (s_0 t)^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0 t\right)\right]^{-1} d\gamma(t),$$

or

$$[\alpha(1) - \alpha(t)] e^{-s_0 t} (s_0 t)^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1,\ s_0 t\right) =$$

$$= \gamma(t) + \gamma(t_1) e^{-s_0 t} (s_0 t)^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0 t\right) \times$$

$$\times \int_t^1 \frac{d}{dt}\left[ e^{-s_0 t} (s_0 t)^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0 t\right)\right]^{-1} dt \quad (t < t_1 < 1).$$

$$= \gamma(t) + \gamma(t_1) \left[ \frac{e^{-s_0 t} (s_0 t)^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0 t\right)}{e^{-s_0} s_0^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0\right)} - 1 \right].$$

Therefore

$$[\alpha(1) - \alpha(t)] e^{-s_0 t} (s_0 t)^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0 t\right) \sim$$

$$\sim \gamma(0+) - \gamma(0+) \qquad (t \to 0+)$$

$$= o(1) \qquad (t \to 0+),$$

since

$$e^{-s_0 t} (s_0 t)^{\frac{1}{2}-k+m} \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0 t\right)$$

tends to zero as $t$ tends to zero under the conditions stated.

Now from (2.3) we have

$$(2.14) \qquad \Psi\left(\frac{1}{2} - k + m,\ 2m+1;\ s_0 t\right) \sim \frac{\Gamma(2m)}{\Gamma\left(\frac{1}{2} - k + m\right)} (s_0 t)^{-2m} \quad (t \to 0)$$

when $\frac{1}{2} - k \pm m \neq 0$ or a negative integer, $\operatorname{Re} m \geqq 0$ $(m \neq 0)$ and $\operatorname{Re}\left(-m - k + \frac{1}{2}\right) > 0$.

Then

$$\lim_{t \to 0} [\alpha(1) - \alpha(t)] e^{-s_0 t} (s_0 t)^{\frac{1}{2} - k + m} \Psi\left(\frac{1}{2} - k + m,\ 2m + 1;\ s_0 t\right)$$

$$= \frac{\Gamma(2m) s_0^{\frac{1}{2} - k + m}}{\Gamma\left(\frac{1}{2} - k + m\right)} \cdot \lim_{t \to 0} \frac{\alpha(1) - \alpha(t)}{t^{k + m - \frac{1}{2}}}.$$

Therefore

$$\alpha(1) - \alpha(t) = o(t^{k + m - \frac{1}{2}}) \qquad (t \to 0)$$

from which (2.13a) follows.

Similarly we can prove that

$$\lim_{t \to 0} \frac{\alpha(t)}{t^{k - m - \frac{1}{2}}} = 0$$

when $\frac{1}{2} - k \pm m \neq 0$ or a negative integer, $\operatorname{Re} m < 0$ and $\operatorname{Re}\left(-k + m + \frac{1}{2}\right) > 0$; and

$$\lim_{t \to 0} \frac{\alpha(t)}{(t^{k - \frac{1}{2}} / \log t)} = 0$$

when $\frac{1}{2} - k \pm m \neq 0$ or a negative integer and $m = 0$.

Thus the theorem is proved.

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