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Composition Operators on Banach Spaces of Formal Power Series.

B. Yousefi - S. Jahedi

dedicated to the memory of Karim Seddighi

Sunto. – *Supponiamo che* $\{\beta(n)\}_{n=0}^{\infty}$ *sia una successione di numeri positivi e* $1 \leq p < \infty$. *Consideriamo lo spazio* $H^p(\beta)$ *di tutte le serie di potenze* $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$, *tali che* $\sum_{n=0}^{\infty} |\hat{f}(n)|^p \beta(n)^p < \infty$. *Supponiamo che* $\frac{1}{p} + \frac{1}{q} = 1$ *e* $\sum_{n=1}^{\infty} \frac{n^{qj}}{\beta(n)^q} = \infty$ *per un intero non-negativo* j. *Dimostriamo che se* C_ϕ *è compatto su* $H^p(\beta)$, *allora il limite non-tangenziale di* $\phi^{(j+1)}$ *ha modulo maggiore di uno, in ogni punto della frontiera del disco unitario aperto. Dimostriamo anche che se* C_ϕ *è di Fredholm su* $H^p(\beta)$, *allora* φ *deve essere un automorfismo del disco unitario aperto.*

Summary. – *Let* $\{\beta(n)\}_{n=0}^{\infty}$ *be a sequence of positive numbers and* $1 \leq p < \infty$. *We consider the space* $H^p(\beta)$ *of all power series* $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$ *such that* $\sum_{n=0}^{\infty} |\hat{f}(n)|^p \beta(n)^p < \infty$. *Suppose that* $\frac{1}{p} + \frac{1}{q} = 1$ *and* $\sum_{n=1}^{\infty} \frac{n^{qj}}{\beta(n)^q} = \infty$ *for some non-negative integer* j. *We show that if* C_φ *is compact on* $H^p(\beta)$, *then the non-tangential limit of* $\varphi^{(j+1)}$ *has modulus greater than one at each boundary point of the open unit disc. Also we show that if* C_φ *is Fredholm on* $H^p(\beta)$, *then* φ *must be an automorphism of the open unit disc.*

Introduction.

First in the following, we generalize the defintions coming in [5].

Let $\{\beta(n)\}$ be a sequence of positive numbers with $\beta(0)=1$ and $1 \leq p < \infty$. We consider the space of sequences $f = \{\hat{f}(n)\}_{n=0}^{\infty}$ such that

$$\|f\|^p = \|f\|_\beta^p = \sum_{n=0}^{\infty} |\hat{f}(n)|^p \beta(n)^p < \infty .$$

The notation $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$ shall be used whether or not the series converges for any value of z. These are called formal power series. Let $H^p(\beta)$ denotes the space of such formal power series. These are reflexive Banach spaces with the norm $\|\cdot\|_\beta$ ([4]) and the dual of $H^p(\beta)$ is $H^q(\beta^{p/q})$ where $\frac{1}{p} +$

$\frac{1}{q} = 1$ and $\beta^{p/q} = \{\beta(n)^{p/q}\}_n$ ([6]). Also if $g(z) = \sum_{n=0}^{\infty} \widehat{g}(n)\, z^n \in H^q(\beta^{p/q})$, then $\|g\|^q = \sum_{n=0}^{\infty} |\, \widehat{g}(n)\,|^q \beta(n)^p$. The Hardy, Bergman and Dirichlet spaces can be viewed in this way when $p = 2$ and respectively $\beta(n) = 1$, $\beta(n) = (n+1)^{-1/2}$ and $\beta(n) = (n+1)^{1/2}$. If $\lim_n \frac{\beta(n+1)}{\beta(n)} = 1$ or $\liminf_n \beta(n)^{1/n} = 1$, then $H^p(\beta)$ consists of functions analytic on the open unit disc U. It is convenient and helpful to introduce the notation $\langle f, g\rangle$ to stand for $g(f)$ where $f \in H^p(\beta)$ and $g \in H^p(\beta)^*$. Note that $\langle f, g\rangle = \sum_{n=0}^{\infty} \widehat{f}(n)\overline{g(n)}\beta(n)^p$. Let $\widehat{f}_k(n) = \delta_k(n)$. So $f_k(z) = z^k$ and then $\{f_k\}_k$ is a basis such that $\|f_k\| = \beta(k)$. Clearly M_z, the operator of multiplication by z on $H^p(\beta)$ shifts the basis $\{f_k\}_k$.

Remember that a complex number λ is said to be a bounded point evaluation on $H^p(\beta)$ if the functional of point evaluation at λ, e_λ, is bounded. The functional of evaluation of the j-th derivative at λ is denoted by $e_\lambda^{(j)}$.

The function φ in $H^p(\beta)$ that maps the unit disc U into itself induces a composition operator C_φ on $H^p(\beta)$ defined by $C_\varphi f = f \circ \varphi$. The operator C_φ is Fredholm, if it is invertible modulo the compact operators. If C_φ is a bounded invertible operator, then φ must be an automorphism of U, that is a one to one map of U onto U.

We say an analytic self-map φ of U has an angular derivative at $w \in \partial U$, if for some $\eta \in \partial U$ the non-tangential limit of $\frac{\varphi(z) - \eta}{z - w}$ when $z \to w$, exists and is finite. We call this limit the angular derivative of φ at w and denoted it by $\varphi'(w)$.

Main results.

We suppose that $H^p(\beta)$ consists of functions analytic on the open unit disc U. We study the Fredholm composition operator C_φ and investigate the compactness and essential norm of C_φ acting on the Banach space $H^p(\beta)$.

LEMMA 1. – Let X be a Banach space of analytic functions on a domain Ω in $\mathbb{C}$. If there exists a sequence of functions g_k in the dual space X^* such that $\|g_k\| = 1$ and $g_k \to 0$ weakly with $\|C_\varphi^*(g_k)\| \to 0$, then C_φ is not Fredholm on X.

PROOF. – Suppose S is any bounded operator on X^*. Then by the hypothesis $\|SC_\varphi^*(g_k)\| \leq \|S\|\|C_\varphi^*(g_k)\| \to 0$ as $k \to \infty$. Now let Q be an arbitrary compact operator on X^*. Since Q is necessarily completely continuous, then we have $\|Q(g_k)\| \to 0$ ([2, p. 177, Proposition 3.3]). Thus $\|(I + Q)\, g_k\| \to 1$ for every compact operator Q on X^*. This implies that $SC_\varphi^* - I$ can not be compact, since else it should be $\|(I + (SC_\varphi^* - I))g_k\| \to 1$ that is a contradiction. Thus C_φ^*, and hence C_φ, is not Fredholm. ■

In the following we use the fact that $e_w \in H^q(\beta^{p/q})$ and $\|e_w\|^q = \sum_{n=0}^{\infty} \frac{|w|^{nq}}{\beta(n)^q} < \infty$ for all w in U ([6]).

THEOREM 2. – Let $\frac{1}{p} + \frac{1}{q} = 1$ and $\sum_{n=0}^{\infty} \frac{n^{qj}}{\beta(n)^q} = \infty$ for some non-negative integer j. If C_φ is Fredholm on $H^p(\beta)$, then φ is an automorphism of the disc.

PROOF. – It is well known that if C_φ is Fredholm, then φ is univalent since else the kernel of C_φ^* will contain an infinite linearly independent set whose elements are differences of evaluation functionals. This is a contradiction, since $\dim \ker C_\varphi^* < \infty$. So we need only show that φ maps U onto U. If not, there exists $v \in \partial\varphi(U) \cap U$ and $z_k \in U$ such that $\varphi(z_k) \to v$. By the Open Mapping Theorem it should be $|z_k| \to 1$.

Let j be the least non-negative integer such that the sum $\sum_{n \geq 0} \frac{n^{qj}}{\beta(n)^q} = \infty$. If $j = 0$, set $e_k = \frac{e_{z_k}}{\|e_{z_k}\|}$. Then $\|e_k\| = 1$. But

$$\lim_k \|e_{z_k}\|^q = \lim_k \sum_{n \geq 0} \frac{|z_k|^{nq}}{\beta(n)^q} = \sum_{n \geq 0} \frac{1}{\beta(n)^q} = \infty$$

and so if p is a polynomial in $H^p(\beta)$, then $\lim_k \langle p, e_k \rangle = \lim_k \frac{p(z_k)}{\|e_{z_k}\|} = 0$. But polynomials are dense in $H^p(\beta)$, thus $e_k \to 0$ weakly as $k \to \infty$. Since v is in U and $\varphi(z_k) \to v$, we have $e_\varphi(z_k) \to e_v$. Since we also have $\|e_{z_k}\| \to \infty$, we conclude that $\|C_\varphi^* e_{z_k}\| = \|e_{\varphi(z_k)}\| / \|e_{z_k}\|$ tends to zero. So by Lemma 1, C_φ is not Fredholm that is a contradiction.

If $j > 0$, let $e_k = \frac{e_{z_k}^{(j)}}{\|e_{z_k}^{(j)}\|}$ where $e_{z_k}^{(j)}$ is the functional of evaluation of the j-th derivative at z_k. Note that $e_w(z) = \sum_{n=0}^{\infty} \frac{1}{\beta(n)^p} \overline{w}^n z^n$ and $e_w^{(j)} = \frac{d^j}{d\overline{w}^j} e_w$. Thus

$$e_{z_k}^{(j)} = \sum_{n=0}^{\infty} n(n-1)(n-2)\ldots(n-j+1) \frac{(\overline{z}_k)^{n-j}}{\beta(n)^p} z^n.$$

Since $|z_k| \to 1$ and $\sum_{n \geq 0} \frac{n^{jq}}{\beta(n)^p} = \infty$, we have

$$\lim_k \|e_{z_k}^{(j)}\|^q = \lim_k \sum_{n=0}^{\infty} (n(n-1)\ldots(n-j+1))^q \frac{|z_k|^{(n-j)q}}{\beta(n)^q} = \infty.$$

Since polynomials are dense in $H^p(\beta)$, by the same manner as in the previous case, we can see that $e_k \to 0$ weakly as $k \to \infty$. Now we show that $\|C_\varphi^* e_k\| \to 0$ as

$k \to \infty$. A straightforward computation gives the following equalities:

$$
\begin{aligned}
&C_\varphi^* e_{z_k}^{(1)} = \varphi'(z_k)\, e_{\varphi(z_k)}^{(1)}\\
&C_\varphi^* e_{z_k}^{(2)} = \varphi'(z_k)^2 + e_{\varphi(z_k)}^{(2)} + \varphi''(z_k)\, e_{\varphi(z_k)}^{(1)}\\
&C_\varphi^* e_{z_k}^{(3)} = \varphi'(z_k)^3\, e_{\varphi(z)}^{(3)} + \varphi'''(z_k)\, e_{\varphi(z_k)}^{(1)} + 2\varphi''(z_k)\, e_{\varphi(z_k)}^{(2)} + \varphi'(z_k)\,\varphi''(z_k)\, e_{\varphi(z_k)}^{(2)}\\
&\qquad\vdots\\
&C_\varphi^* e_{z_k}^{(j)} = \varphi'(z_k)^j\, e_{\varphi(z_k)}^{(j)} + \varphi^{(j)}(z_k)\, e_{\varphi(z_k)}^{(1)} + \text{lower order terms}
\end{aligned}
$$

where the lower order terms involves functionals of evaluation of derivatives of order less than j at $\varphi(z_k)$ with coefficients involving products of derivatives of φ at z_k of order less than j. From this it follows that $\|C_\varphi^* e_k\| \to 0$ as $k \to 0$. To see this first suppose that $j = 1$. Thus we have

$$C_\varphi^* e_k = \frac{\varphi'(z_k)\, e_{\varphi(z_k)}^{(1)}}{\|e_{z_k}^{(1)}\|} = \langle \varphi, e_k \rangle\, e_{\varphi(z_k)}^{(1)}.$$

But $\varphi(z_k) \to \nu$, where $\nu \in U$. So $\|e_{\varphi(z_k)}^{(1)}\| \to \|e_\nu^{(1)}\| < \infty$. Also since $e_k \to 0$ weakly, $\langle \varphi, e_k \rangle \to 0$ as $k \to \infty$. Thus indeed $\|C_\varphi^* e_k\| \to 0$ as $k \to \infty$.

If $j > 1$, remark that for all $i < j$ we have

$$e_{\varphi(z_k)}^{(i)} = \sum_{n=1}^{\infty} \frac{n!}{(n-i)!} \frac{(\overline{\varphi(z_k)})^{n-i}}{\beta(n)^p}$$

and so

$$
\begin{aligned}
\|e_{\varphi(z_k)}^{(i)}\|^q &= \sum_{n=i}^{\infty} (n(n-1)\dots(n-i+1))^q \frac{|v(z_k)|^{n-i}}{\beta(n)^q}\\
&\leqslant \sum_{n=i}^{\infty} \frac{n^{iq}}{\beta(n)^q} \leqslant \sum_{n=i}^{\infty} \frac{n^{(j-1)q}}{\beta(n)^q} < \infty,
\end{aligned}
$$

since j is the least non-negative integer such that $\sum_{n=0}^{\infty} \frac{n^{jq}}{\beta(n)^q} = \infty$. Thus the limit of the norms of the functionals of evaluation of derivatives at $\varphi(z_k)$ of order less than j remain bounded in U. Also, by the Principle of Uniform Boundedness Theorem $\sup_k \|e_{z_k}^{(i)}\| < \infty$ for $i < j$ and all derivatives of φ at z_k of order less than j are bounded. Note that $\|e_{z_k}^{(j)}\| \to \infty$ and $\varphi(z_k) \to \nu \in U$. Thus we have $\lim_{k\to\infty} \|C_\varphi^* e_k\| = 0$ provided that

$$\lim_{k\to\infty} \frac{1}{\|e_{z_k}^{(j)}\|} \|(\varphi'(z_k))^j\, e_{\varphi(z_k)}^{(j)} + \varphi^{(j)}(z_k)\, e_{\varphi(z_k)}^{(1)}\| = 0 .$$

Clearly

$$(*)\qquad \frac{1}{\|e_{z_k}^{(j)}\|}\|(\varphi'(z_k))^j e_{\varphi(z_k)}^{(j)} + \varphi^{(j)}(z_k)\, e_{\varphi(z_k)}^{(1)}\| \leq$$

$$\frac{|\varphi'(z_k)|^j}{\|e_{z_k}^{(j)}\|}\|e_{\varphi(z_k)}^{(j)}\| + \frac{|\varphi^{(j)}(z_k)|}{\|e_{z_k}^{(j)}\|}\|e_{\varphi(z_k)}^{(1)}\| \leq$$

$$\frac{\|\varphi\|_{H^p(\beta)}^j \|e_{z_k}^{(1)}\|^j}{\|e_{z_k}^{(j)}\|}\|e_{\varphi(z_k)}^{(j)}\| + |\langle\varphi, e_k\rangle| \,.\, \|e_{\varphi(z_k)}^{(1)}\|.$$

Note that $\|e_{z_k}^{(j)}\| \to \infty$ and $\lim_k \|e_{z_k}^{(1)}\| < \infty$, since $1 < j$. Also $\|e_{\varphi(z_k)}^{(i)}\| \to \|e_\nu^{(i)}\| < \infty$ for $i = 1, j$ and $\langle \varphi, e_k\rangle \to 0$, since $e_k \to 0$ weakly. Thus indeed the term in $(*)$ tends to zero as $k \to \infty$ and so $\|C_\varphi^* e_k\| \to 0$ which by the lemma implies that C_φ is not Fredholm that is a contradiction. ∎

Note that by the Julia Caratheodory Theorem ([3]), φ has an angular derivative at $w \in \partial U$ if and only if φ' has non-tangential limit at w, and φ has non-tangential limit of modulus one at w. Consider the open Euclidean disc, Julia disc, $J(\xi, a) = \{z \in U;\ |\xi - z|^2 < a(1 - |z|^2)\}$ of radius $\frac{a}{1+a}$ and center at $\frac{\xi}{1+a}$, whose boundary is tangent to ∂U at ξ. By the Julia's Lemma ([1]), if $\xi \in \partial U$ and φ is an analytic function such that $B_\varphi = \inf_{\xi \in \partial U} |\varphi'(\xi)| < \infty$, then $\varphi(J(\xi, a)) \subseteq J(\varphi(\xi), aB_\varphi)$.

Recall that the essential norm of C_φ is denoted by $\|C_\varphi\|_e$ and is the distance in the operator norm from C_φ to the compact operators.

THEOREM 3. – Let $\frac{1}{p} + \frac{1}{q} = 1$ and $\sum_{n=1}^{\infty} \frac{n^{qj}}{\beta(n)^q} = +\infty$ for some non-negative integer j. Also for $0 \leq i \leq j$ let $\varphi^{(i)}$ be an analytic self map of the unit disc U. If C_φ is a bounded operator on $H^p(\beta)$ and $|\varphi^{(j+1)}(\xi)| \leq 1$ for some $\xi \in \partial U$, then $\|C_\varphi\|_e \geq 1$ and C_φ is not compact.

PROOF. – Let $\{z_k\}$ be any sequence in U with $z_k \to \xi$. Also let j be the least non-negative integer such that the sum $\sum_{n=1}^{\infty} \frac{n^{qj}}{\beta(n)^q} = +\infty$. Set $e_k = \frac{e_{z_k}^{(j)}}{\|e_{z_k}^{(j)}\|}$. Then $\|e_k\| = 1$ and by the same method used in the proof of Theorem 2, $e_k \to 0$ weakly as $k \to \infty$. If K is any compact operator, then K^* is completely continuous and since $e_k \to 0$ weakly, it should be $\|K^* e_k\| \to 0$. By definition $\|C_\varphi\|_e = \inf\{\|C_\varphi - K\| : K \text{ is compact}\}$ and

$$\|C_\varphi - K\| = \|(C_\varphi - K)^*\| \geq \|(C_\varphi - K)^* e_k\| \geq \|C_\varphi^* e_k\| - \|K^* e_k\|.$$

If $k \to \infty$, then since $\|K^* e_k\| \to 0$, we have $\|C_\varphi\|_e \geq \overline{\lim_k} \|C_\varphi^* e_k\|$. Now we show that

$$\overline{\lim_k} \|C_\varphi^* e_k\| = \overline{\lim_k} \frac{\|e_{\varphi(z_k)}^{(j)}\|}{\|e_{z_k}^{(j)}\|}.$$

Note that since $|\varphi^{(j+1)}(\xi)| \leq 1$, by the Julia's Caratheodory theorem the non-tangential limit of $\varphi^{(i)}(\xi)$ have modulus one for $i = 0, 1, \ldots, j$.

If $j = 0$, then $e_k = \frac{e_{z_k}}{\|e_{z_k}\|}$ and $C_\varphi^* e_k = \frac{e_{\varphi(z_k)}}{\|e_{z_k}\|}$. If $j = 1$, then $e_k = \frac{e_{z_k}^{(1)}}{\|e_{z_k}^{(1)}\|}$ and $C_\varphi^* e_k = \varphi'(z_k) \frac{e_{\varphi(z_k)}^{(1)}}{\|e_{z_k}^{(1)}\|}$. But the non-tangential limit of $\varphi'(\xi)$ has modulus one and so $\overline{\lim_k} \|C_\varphi^* e_k\| = \overline{\lim_k} \|e_{\varphi(z_k)}^{(1)}\| / \|e_{z_k}^{(1)}\|$.

If $j > 1$, then $e_k = e_{z_k}^{(j)} / \|e_{z_k}^{(j)}\|$ and

$$C_\varphi^* e_k = \frac{1}{\|e_{z_k}^{(j)}\|} (\varphi'(z_k)^j e_{\varphi(z_k)}^{(j)} + L_{j,k})$$

where $L_{j,k}$ is the sum of lower order terms and involves derivatives of order less than j at $\varphi(z_k)$, i.e., terms of the type $e_{\varphi(z_k)}^{(i)}$ $(i < j)$, with coefficients involving product of derivatives of φ at z_k of order less than or equal to j. Remark that since j is the least non-negative integer such that $\sum_{n=0}^{\infty} \frac{n^{jq}}{\beta(n)^q} = +\infty$, then we have

$$\|e_{\varphi(z_k)}^{(i)}\|^q = \sum_{n=1}^{\infty} (n(n-1)\ldots(n-i+1))^q \frac{|\varphi(z_k)|^{n-i}}{\beta(n)^q} \leq \sum_{n=1}^{\infty} \frac{n^{(j-1)q}}{\beta(n)^q} < \infty$$

for $i < j$. So $\lim_{k \to \infty} \|e_{\varphi(z_k)}^{(i)}\|$ remains bounded for all i less than j. Also since $\|e_{z_k}^{(j)}\| \to \infty$ and $\varphi^{(i)}(\xi)$ has the non-tangential limit of modulus one for all $i \leq j$, thus indeed $\lim_k \frac{\|L_{j,k}\|}{\|e_{z_k}^{(j)}\|} = 0$.

Therefore

$$\overline{\lim_k} \|C_\varphi^* e_k\| = \overline{\lim_k} |\varphi'(z_k)|^j \frac{\|e_{\varphi(z_k)}^{(j)}\|}{\|e_{z_k}^{(j)}\|}$$

$$= \overline{\lim_k} \frac{\|e_{\varphi(z_k)}^{(j)}\|}{\|e_{z_k}^{(j)}\|}.$$

Now to complete the proof it is sufficient to show that $\overline{\lim_k} \frac{\|e_{\varphi(z_k)}^{(j)}\|}{\|e_{z_k}^{(j)}\|} \geq 1$. For this set $z_k = \left(1 - \frac{1}{k}\right) \xi$. Then there exists a sequence $\{r_k\}$ of non-negative numbers such that z_k is the point on $\partial J(\xi, r_k)$ closest to 0. Therefore by the Julia's Lemma

$$\varphi(z_k) \in \varphi(\partial J(\xi, r_k)) \subseteq \partial\varphi(J(\xi, r_k)) \subseteq \partial J(\varphi(\xi), r_k).$$

It follows that $|\varphi(z_k)| \geqslant |z_k|$ for all k. Now since $\|e_z^{(j)}\|^q = \sum_{n=j}^{\infty} \frac{n!}{(n-j)!} \frac{|z|^{n-j}}{\beta(n)^q}$, the norm $\|e_z^{(j)}\|$ increases with $|z|$. Thus for all k, $\|e_{\varphi(z_k)}^{(j)}\| / \|e_{z_k}^{(j)}\| \geqslant 1$ and indeed $\|C_\varphi\|_e \geqslant 1$. This implies that C_φ is not compact and so the proof is complete. ■

Corollary 4. – Let $\frac{1}{p} + \frac{1}{q} = 1$ and $\sum_{n=1}^{\infty} \frac{n^{qj}}{\beta(n)^q} = +\infty$ for some non-negative integer j. If C_φ is compact on $H^p(\beta)$, then $|\varphi^{(j+1)}(\xi)| > 1$ for all ξ in ∂U such that $\varphi^{(j+1)}(\xi)$ exists.

REFERENCES

[1] L. Ahlfors, *Conformal Invariants*, McGraw-Hill, New York, 1973.

[2] J. B. Conway, *A Course in Functional Analysis*, Springer-Verlag, New York, 1985.

[3] W. Rudin, *Function Theory in the Unit Ball of* $\mathbb{C}^n$, Grundlehren der Mathematischen Wissenschaften, 241, Springer-Verlag, Berlin, 1980.

[4] K. Seddighi - K. Hedayatiyan - B. Yousefi, *Operators acting on certain Banach spaces of analytic functions*, Internat. J. Math. & Math. Sci., **18**, No. 1 (1995), 107-110.

[5] A. L. Shields, *Weighted shift operators and analytic function theory*, Math. Survey, A.M.S. Providence, **13** (1974), 49-128.

[6] B. Yousefi, *On the space* $l^p(\beta)$, Rendiconti del Circolo Matematico di Palermo Serie II, Tomo **XLIX** (2000), 115-120.

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