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A Linear Magnetic Bénard Problem with Tensorial Electrical Conductivity. (*)

A. Georgescu - L. Palese - A. Redaelli

Sunto. – *Si studia, nell'ipotesi che sussista il principio di scambio delle stabilità, il problema agli autovalori che governa la stabilità lineare della quiete per un problema di Bénard elettroanisotropo, in presenza di correnti di Hall e di ion-slip. Si risolvono due problemi agli autovalori dello stesso ordine derivanti dall'aver scomposto le perturbazioni nelle loro parti pari e dispari, espresse come somme di serie di Fourier di opportuni insiemi totali in spazi di Hilbert separabili. Si determinano le curve neutrali applicando il metodo di Budiansky-DiPrima Si prova l'effetto instabilizzante delle correnti elettroanisotrope.*

Summary. – *For normal mode perturbations, in the hypothesis that the principle of exchange of stabilities holds, the eigenvalue problem defining the neutral curves of the linear stability for a magnetic electroanisotropic Bénard problem is solved by Budiansky-DiPrima method. The unknown functions are taken as Fourier series on some total sets of separable Hilbert spaces and the expansion functions satisfied only part of the boundary conditions of the problem. This introduces some constraints to be satisfied by the Fourier coefficients. In order to keep the number of these constraints as low as possible we are lead to use total sets for the even velocity and temperature fields different from the case when velocity and temperature are odd.The splitting of the unknown functions into even and odd parts leads to two problems of the same order as the given one each of which containing even as well as odd order parts of these functions. The secular equations involve series which are truncated to one and two terms, the last situation corresponding to best results. A closed form of the neutral curve is obtained. The presence of the Hall currents is proved to be destabilizing.*

1. – Introduction.

One of the most important rheological parameters in magnetohydrodynamics is the electrical conductivity. If the usual Ohm's law is assumed to hold then it is a

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scalar quantity. If the generalized Ohm's law, i.e.

$$\boldsymbol{j} = \sigma \boldsymbol{E} - \omega_e \tau_e \boldsymbol{j} \times \frac{\boldsymbol{B}}{B} + f^2 \omega_e \tau_e \omega_I \tau_{In} \left[\frac{\boldsymbol{B}}{B} \left(\frac{\boldsymbol{B}}{B} \cdot \boldsymbol{j} \right) - \boldsymbol{j} \right] \tag{1}$$

is assumed, then the conductivity becomes a tensor [31]. In (1) $\boldsymbol{j}$, $\boldsymbol{E}$, $\boldsymbol{B}$ stand for the density current vector, electric field and magnetic field respectively, while σ, ω_e, τ_e, ω_I, τ_{In} and f are the scalar electrical conductivity, the Larmor frequency (i.e. ciclotron frequency of electrons), the mean electron collision time, the frequency of ions, the (average) time of collision of ions with neutral particles and the mass fraction of atoms which are not ionized, respectively.

The equation (1) represents the Ohm's law for a partially ionized fluid. In the case of a fully ionized fluid, i.e. there are no neutral particles, $f = 0$, and equation (1) reduces to

$$\boldsymbol{j} = \sigma \boldsymbol{E} - \omega_e \tau_e \boldsymbol{j} \times \frac{\boldsymbol{B}}{B},$$

where the second term in the right-hand side is the Hall current. The last term in the right-hand side of (1), that is the ion-slip current, becomes important at "small values" of $\omega_e \tau_e$ [31] at which the electron-ion collisions dominate the electron motion. For moderate magnetic fields the Hall current can be neglected, otherwise the tensorial property of the electrical conductivity must be taken into account. In any case, if $\omega_e \tau_e >> 1$, transverse conductivities and, therefore, tensorial electrical conductivity (which can be derived from (1) [20] [31]) must be considered.

The problem of existence, continuous dependence, uniqueness, linear and non linear stability of the thermodiffusive equilibrium for the Bénard problem has been largely investigated in the hydrodynamic case [17] [18], in magnetohydrodynamic case [26], as well as in the presence of Hall and ion-slip currents [1], [4], [5], [8], [9], [12]-[14], [19], [21], [22]-[25], [27]-[30]. In [13], [14] the problem of the linear stability of the thermodiffusive equilibrium for the magnetic Bénard problem in the presence of Hall and ion-slip currents is studied by the Chandrasekhar-Galerkin method ; in [8] it can be found a first numerical estimation of the stabilizing-instabilizing effect of the Hall and ion-slip currents. In these two papers the boundaries are free. In [24], for rigid boundaries, we reformulate the equations governing the stability problem of the thermodiffusive equilibrium for a magnetic Bénard problem in the presence of a Hall current. Then we apply the energy method and solve the associated variational problem by the Budiansky-DiPrima method. It is obtained the destabilizing effect of the Hall current.

In this work , for a horizontal layer with free boundaries, we consider another linear magnetic Bénard problem for a thermoelectrically conducting fluid and study the linear stability of the thermodiffusive equilibrium in the class of the normal mode perturbations. The direct method of Budiansky-DiPrima type of expansion in Fourier series is used.

In Section 2 we formulate the general mathematical problem, in Section 3 we deduce the system in the Fourier coefficients for normal mode perturbations, in Section 4, for even temperature and velocity and odd magnetic fields, we derive some *approximate* neutral curves for the linear instability. They are obtained from the secular equation involving series truncated to the first two terms. In Section 5 we consider the same problem as in Section 4 for odd temperature and velocity fields and even magnetic field. In Section 5.1 we treat the case $\beta_H = 0$. In Section 5.2 we consider the case $\beta_H \neq 0$ and determine the approximate neutral curve obtained by retaining one or two terms in the Fourier series representing the secular equation. We find that the Hall current has a destabilizing effect [8] [13] [14]; in the absence of the Hall current we obtain the smallest eigenvalue leading to the neutral curve.

2. – Mathematical problem.

In the framework of continua and in the domain of validity of the Oberbeck-Boussinesq approximation, consider a homogeneous thermoelectrically conducting fluid with tensorial electrical conductivity. Assume that the fluid is situated in a horizontal layer S bounded by the planes $\pi_0 : z = 0$ and $\pi_1 : z = 1$, both stress-free, perfectly thermally and electrically conductors. Furthermore a constant vertical temperature gradient $\beta > 0$ is maintained, in the presence of a uniform vertical magnetic field $\boldsymbol{H}_0$. The dimensionless equations governing the perturbation $\boldsymbol{u}, \boldsymbol{h}, \theta, p$ of the thermodiffusive equilibrium m_0

$$m_0 \equiv \{\boldsymbol{U} = \boldsymbol{0}, \quad \boldsymbol{H} = H_0 \boldsymbol{k}, \quad T = -\beta z + T_0, \quad p_0 = p_0(z)\}$$

are [2], [20], [31]

$$(1)\qquad \begin{cases} \dfrac{\partial}{\partial t}\boldsymbol{u} = -\boldsymbol{u}\cdot\nabla\boldsymbol{u} - \nabla p + P_m\Delta\boldsymbol{u} + P_m M^2(\boldsymbol{H}_0+\boldsymbol{h})\cdot\nabla\boldsymbol{h} + \mathcal{R}\dfrac{P_m^2}{P_r}\theta\boldsymbol{k}, \\ \dfrac{\partial}{\partial t}\boldsymbol{h} = \nabla\times[\boldsymbol{u}\times(\boldsymbol{H}_0+\boldsymbol{h})] + \Delta\boldsymbol{h} + \beta_H\nabla\times[(\boldsymbol{H}_0+\boldsymbol{h})\times\nabla\times\boldsymbol{h}], \\ \dfrac{\partial}{\partial t}\theta = -\boldsymbol{u}\cdot\nabla\theta + \boldsymbol{u}\cdot\boldsymbol{k} + \dfrac{P_m}{P_r}\Delta\theta, \\ \nabla\cdot\boldsymbol{u} = 0, \\ \nabla\cdot\boldsymbol{h} = 0, \end{cases}$$

where $\boldsymbol{u}$ is the velocity field, $\boldsymbol{h}$ is the magnetic field, $\boldsymbol{k}$ is the upwards positive unit vector, θ is the temperature, p is the pressure, the positive coefficients P_r, P_m, M^2 and R are the Prandtl, Prandtl magnetic, Hartmann and Rayleigh numbers respectively. The Hall coefficient occurring in the generalized Ohm's law was denoted by β_H [20], [31]. For the case of free perfectly conducting walls in the presence of the Hall current the boundary conditions are [30]

$$(2)\quad \begin{cases} \boldsymbol{u}\cdot\boldsymbol{n}=\theta=0 \quad \boldsymbol{n}\times\boldsymbol{D}\cdot\boldsymbol{n}=\boldsymbol{0}, \\ \boldsymbol{h}\cdot\boldsymbol{n}=0 \qquad \{\nabla\times\boldsymbol{h}+\beta_H[\nabla\times\boldsymbol{h}\times(\boldsymbol{H}_0+\boldsymbol{h})]\}\times\boldsymbol{n}=\boldsymbol{0} \quad z=0,1 \quad t\geq 0 \end{cases}$$

where $\boldsymbol{D}$ is the velocity deformation tensor and $\boldsymbol{n}$ is the external normal to the layer boundary. Assume that the perturbation fields are doubly periodic of period $2\pi/a$ and $2\pi/\beta$ in x and y direction respectively and let us use the variables $\boldsymbol{k}\cdot\boldsymbol{u}, \boldsymbol{k}\cdot\nabla\times\boldsymbol{u}, \boldsymbol{k}\cdot\boldsymbol{h}, \boldsymbol{k}\cdot\nabla\times\boldsymbol{h}$ [11]. Denote by $\Omega=\left[0,\frac{2\pi}{a}\right]\times\left[0,\frac{2\pi}{\beta}\right]\times[0,1]$ the periodicity cell. Thus, from the equations (1), linearized about the equilibrium m_0, we obtain

$$(3)\quad \begin{cases} \dfrac{\partial}{\partial t}\Delta w = P_m\Delta\Delta w + P_m M^2\dfrac{\partial}{\partial z}\Delta h_3 + \mathcal{R}\dfrac{P_m^2}{P_r}\Delta_1\theta, \\ \dfrac{\partial}{\partial t}\zeta = P_m\Delta\zeta + P_m M^2\dfrac{\partial}{\partial z}j, \\ \dfrac{\partial}{\partial t}h_3 = \dfrac{\partial}{\partial z}w + \Delta h_3 - \beta_H\dfrac{\partial}{\partial z}j, \\ \dfrac{\partial}{\partial t}j = \dfrac{\partial}{\partial z}\zeta + \Delta j + \beta_H\dfrac{\partial}{\partial z}\Delta h_3, \\ \dfrac{\partial}{\partial t}\theta = w + \dfrac{P_m}{P_r}\Delta\theta, \end{cases}$$

where $\quad w=\boldsymbol{k}\cdot\boldsymbol{u}, \quad h_3=\boldsymbol{k}\cdot\boldsymbol{h}, \quad \zeta=\boldsymbol{k}\cdot\nabla\times\boldsymbol{u}, \quad j=\boldsymbol{k}\cdot\nabla\times\boldsymbol{h}, \quad \Delta_1=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}.$ Equations (3_1), (3_2) and (3_4) are obtained by applying [2] the operators $\boldsymbol{k}\cdot\nabla\times\nabla\times$, $\boldsymbol{k}\cdot\nabla\times$ to the equation (1_1) and $\boldsymbol{k}\cdot\nabla\times$ to the equation (1_2) respectively. The boundary conditions obtained by linearizing (2) are

$$(4)\qquad w=\frac{\partial^2}{\partial z^2}w=h_3=\frac{\partial}{\partial z}j=\frac{\partial}{\partial z}\zeta=\theta=0 \qquad \text{at} \qquad z=0,1.$$

In spite of their similarity the problem (3)-(4) in this paper and the problem (2)-(4) in [24] are different due to the boundary conditions. This will imply notable differences in the solution and solution method. Assume that the perturbations are normal modes, i.e.

$$(5)\qquad (w,h_3,j,\zeta,\theta)=\{W(z),K(z),X(z),Z(z),\Theta(z)\exp\left[i(ax+\beta y)+\sigma t\right],$$

introduce (5) into (3), (4) and perform the change of variables $z \to z - 0.5$ to get

$$(6) \quad \begin{cases} (D^2 - a^2 - \sigma)K + DW - \beta_H DX = 0, \\ [P_m(D^2 - a^2) - \sigma]Z + P_m M^2 DX = 0, \\ (D^2 - a^2)[P_m(D^2 - a^2) - \sigma]W + P_m M^2 D(D^2 - a^2)K - \mathcal{R}\dfrac{P_m^2}{P_r} a^2 \Theta = 0, \\ (D^2 - a^2 - \sigma)X + DZ + \beta_H D(D^2 - a^2)K = 0, \\ \left[\dfrac{P_m}{P_r}(D^2 - a^2) - \sigma\right]\Theta + W = 0, \end{cases}$$

where $a^2 = \alpha^2 + \beta^2$,

$$(7) \qquad W = D^2 W = K = DX = DZ = \Theta = 0 \qquad z = \pm 0.5.$$

3. – Series solutions for the eigenvalue problem.

If the instability sets in as a stationary convection, then in (6) we must take $\sigma = 0$. In the following we consider only this case. Therefore consider the eigenvalue problem (6) (7) with $\sigma = 0$. It is not easy to find its eigensolutions by using total sets in $L^2(-0.5, 0.5)$ of orthonormal functions satisfying all boundary conditions and being quite simple [3], [6]. This is why we expand the unknown functions in Fourier series on very simple expansion functions, namely sinus and cosinus. However the last functions do not satisfy all boundary conditions. More precisely, let us write the unknown functions as the sum of their even and odd part, e.g.

$$W = W^e + W^0 = \sum_{n=1}^{\infty} W^e_{2n-1} E_{2n-1} + \sum_{n=1}^{\infty} W^o_{2n-1} F_{2n-1},$$

where $E_{2n-1} = \sqrt{2}\cos([(2n-1)\pi z)]$ and $F_{2n-1} = \sqrt{2}\sin([(2n-1)\pi z)]$.

The sets $\{E_1, E_3, E_5, \cdots\}$, and $\{F_1, F_3, F_5, \cdots\}$ are total in the subsets of even and odd functions of $L^2(-0.5, 0.5)$ respectively [3], [6]. The boundary conditions $W(\pm 1/2) = 0$ imply $W^e(1/2) + W^o(1/2) = 0$, $W^e(-1/2) + W^o(-1/2) = 0$ and, taking into account that $W^e(1/2) = W^e(-1/2)$ and $W^o(-1/2) = -W^o(1/2)$, it follows that $W^e(\pm 1/2) = 0$ and $W^o(\pm 1/2) = 0$. Similar relation hold for all other unknown functions. Then, recalling that the derivative of an even function is a odd function, (7) reads

$$(7') \qquad W^e = D^2 W^e = K^o = DX^e = DZ^o = \Theta^e = 0 \qquad z = \pm 0.5$$

$$(7'') \qquad W^o = D^2 W^0 = K^e = DX^o = DZ^e = \Theta^o = 0 \qquad z = \pm 0.5.$$

Taking into account that the equality of two functions, namely one odd and the other even, implies that both functions are equal to zero, it follows that (6) decomposes into two systems: one in W^e, Θ^e, Z^o, X^e and K^o

$$(6')\qquad \begin{cases} (D^2-a^2)K^o + DW^e - \beta_H DX^e = 0, \\ (D^2-a^2)Z^o + M^2 DX^e = 0, \\ (D^2-a^2)^2 W^e + M^2 D(D^2-a^2)K^o - \mathcal{R}a^2 \dfrac{P_m}{P_r}\Theta^e = 0, \\ (D^2-a^2)X^e + DZ^o + \beta_H D(D^2-a^2)K^o = 0, \\ \dfrac{P_m}{P_r}(D^2-a^2)\Theta^e + W^e = 0 \end{cases}$$

and the other in W^o, Θ^o, Z^e, X^o and K^e

$$(6'')\qquad \begin{cases} (D^2-a^2)K^e + DW^o - \beta_H DX^o = 0, \\ (D^2-a^2)Z^e + M^2 DX^o = 0, \\ (D^2-a^2)^2 W^o + M^2 D(D^2-a^2)K^e - \mathcal{R}a^2 \dfrac{P_m}{P_r}\Theta^o = 0, \\ (D^2-a^2)X^o + DZ^e + \beta_H D(D^2-a^2)K^e = 0, \\ \dfrac{P_m}{P_r}(D^2-a^2)\Theta^o + W^o = 0. \end{cases}$$

With (6′) (resp (6″)) we associate the boundary conditions (7′) (resp (7″)).

The Fourier coefficients of the derivatives of the unknown functions can be immediately obtained by taking into account the boundary conditions in the general formulae [6]. Namely, let $U^e : [-0.5, 0.5] \to \boldsymbol{R}$ be an even function. Then, by backward integration technique, the derivatives of its expansion in Fourier series upon the set $\{E_1, E_2, \cdot\}$, i.e. $U^e(z) = \sum_{n=1}^{\infty} U^e_{2n-1} E_{2n-1}(z)$, read

$$D^{2k+1}U^e(z) = \sum_{n=1}^{\infty} U^{e(2k+1)}_{2n-1} F_{2n-1}(z), \qquad D^{2k}U^e(z) = \sum_{n=1}^{\infty} U^{e(2k)}_{2n-1} E_{2n-1}(z),$$

where

$$U^{e(2k+1)}_{2n-1} = 2\sqrt{2}(-1)^{n+1} D^{2k} U^e(0.5) - (2n-1)\pi U^{e(2k)}_{2n-1},$$

$$U^{e(2k)}_{2n-1} = (2n-1)\pi U^{e(2k-1)}_{2n-1}.$$

Let $U^o : [-0.5, 0.5] \to \boldsymbol{R}$ be an odd function. Then, by the same technique, the derivatives of its expansion in Fourier series upon the set $\{F_1, F_2, \cdot\}$, namely $U^o(z) = \sum_{n=1}^{\infty} U^o_{2n-1} F_{2n-1}(z)$, read

$$D^{2k+1}U^o(z) = \sum_{n=1}^{\infty} U^{o(2k+1)}_{2n-1} E_{2n-1}(z), \qquad D^{2k}U^o(z) = \sum_{n=1}^{\infty} U^{o(2k)}_{2n-1} F_{2n-1}(z),$$

where

$$U^{o(2k+1)}_{2n-1} = (2n-1)\pi U^{o(2k)}_{2n-1}, \quad U^{o(2k)}_{2n-1} = 2\sqrt{2}(-1)^{n+1} D^{2k-1} U^o(0.5) - (2n-1)\pi U^{o(2k-1)}_{2n-1}.$$

4. – Neutral curves for the even case.

The case where the velocity and temperature are even functions of z is referred to as the *even case*. Then, by using the notation $A_n = (2n-1)^2\pi^2 + a^2$, problem (6′) (7′) becomes

$$\text{(8)} \quad \begin{cases} -A_n K^o_{2n-1} - (2n-1)\pi W^e_{2n-1} + \beta_H (2n-1)\pi X^e_{2n-1} = \\ \qquad = 2\sqrt{2}(-1)^n [a_6 - \beta_H a_4], \\ -A_n Z^o_{2n-1} - M^2 (2n-1)\pi X^e_{2n-1} = 2\sqrt{2}(-1)^n a_4 M^2, \\ A_n^2 W^e_{2n-1} - M^2 A_n (2n-1)\pi K^o_{2n-1} - \dfrac{P_m}{P_r} Ra^2 \Theta^e_{2n-1} = \\ \qquad = 2\sqrt{2}(-1)^n (2n-1)\pi a_6 M^2, \\ -A_n X^e_{2n-1} + (2n-1)\pi Z^o_{2n-1} - \beta_H (2n-1)\pi A_n K^o_{2n-1} = \\ \qquad = 2\sqrt{2}(-1)^n (2n-1)\pi [a_6 \beta_H + a_4], \\ -A_n \dfrac{P_m}{P_r} \Theta^e_{2n-1} + W^e_{2n-1} = 0, \end{cases}$$

where $a_6 = DK^o(0.5)$, $a_4 = X^e(0.5)$ and the constraints read

$$\sum_{n=1}^{\infty} [2\sqrt{2} a_4 - (-1)^{n+1} (2n-1)\pi X^e_{2n-1}] = 0, \qquad \sum_{n=1}^{\infty} (-1)^{n+1} K^o_{2n-1} = 0.$$

Denoting $X^e_{2n-1} = \Delta_{4n}/\Delta_n$, $K^o_{2n-1} = \Delta_{5n}/\Delta_n$, we obtain

$$\Delta_n = \frac{P_m}{P_r} A_n \Big\{ (Ra^2 - A_n^3) \big[A_n^2 + (A_n - a^2)(M^2 + A_n \beta_H^2) \big]$$

$$- M^2 A_n (A_n - a^2) \big[A_n^2 + (A_n - a^2) M^2 \big] \Big\},$$

$$\Delta_{4n} = 2\sqrt{2}(-1)^n \frac{P_m}{P_r} (2n-1)\pi a_4 \Big[(A_n^3 - Ra^2)(A_n^2 + M^2 A_n + A_n^2 \beta_H^2)$$

$$+ M^2 A_n^3 (A_n - a^2) + M^4 A_n^2 (A_n - a^2) \Big],$$

$$\Delta_{5n} = 2\sqrt{2}(-1)^n \frac{P_m}{P_r} \Big\{ a_4 a^2 \beta_H A_n (Ra^2 - A_n^3) \Big\} - 2\sqrt{2}(-1)^n \frac{\Delta_n}{A_n} a_6.$$

Then the restrictions imply the following secular equation, yielding the eigenvalues R as functions of a^2 and physical parameters M and β_H

$$\sum_{n=1}^{\infty}\frac{1}{A_n}\cdot\sum_{n=1}^{\infty}\left[\frac{a^2A_n^2(Ra^2-A_n^3)-a^2M^2A_n^3(A_n-a^2)}{\Delta_n}\right]=0.$$

Introduce the notation

(9) $\quad H_n=\left[A_n^2+(A_n-a^2)M^2\right],\quad L_n=A_n(A_n-a^2),\quad X_n=Ra^2-H_nA_n.$

Then $\Delta_n=A_n(P_m/P_r)[X_n(H_n+\beta_H^2L_n)+M^2\beta_H^2L_n^2]$ such that this equation becomes

$$\sum_{n=1}^{\infty}\frac{A_nX_n}{X_n(H_n+\beta_H^2L_n)+M^2\beta_H^2L_n^2}=0. \tag{10}$$

If the Hall effect is absent, i.e. $\beta_H=0$, the functions in (10) are singular at $X_n=0$, $n=1,2,\cdots$. Then, assuming that $X_n\neq 0$ it follows that (10) fails to represent the secular equation. Therefore we are forced to consider this case separately. A treatment similar to that in Section 5.1 shows that, in fact, the solutions of the true secular equation are $X_n=0$, $n=1,2,\cdots$ and they correspond to the eigenvalues $Ra^2=H_nA_n$, or, equivalently, $R=\dfrac{A_n^3+(A_n-a^2)M^2A_n}{a^2}$. The smallest eigenvalue $R=\dfrac{(\pi^2+a^2)^3+M^2\pi^2(\pi^2+a^2)}{a^2}$ defining the neutral curve corresponds to $n=1$ and it is equal to IV, (163) of [2].

If $\beta_H\neq 0$ and $n=1$ the eigenvalue is still that from the case $\beta_H=0$, i.e. $R=H_1A_1a^{-2}$, and it corresponds to $X_1=0$. If $\beta_H\neq 0$ and $n=1$ and 2, from (10) we derive the equation in X_2

$$X_2^2+X_2[Q+M^2\beta_H^2B_+]+M^2\beta_H^2QD=0,$$

where $Q=X_1-X_2\equiv H_2A_2-H_1A_1$, the solutions of which are

$$X_2^{(1),(2)}=\frac{-(Q+M^2\beta_H^2B_+)\pm\sqrt{(Q+M^2\beta_H^2B_-)^2+4M^4\beta_H^4C}}{2}, \tag{11}$$

where $B_\pm=(A_2L_1^2\pm A_1L_2^2)(A_1G_2+A_2G_1)^{-1}$, $C=A_1A_2L_1^2L_2^2(A_1G_2+A_2G_1)^{-1}$, $D=A_1L_2^2(A_1G_2+A_2G_1)^{-1}$, $G_n=A_n^2+(A_n-a^2)(M^2+A_n\beta_H^2)$, whence the result

THEOREM 1. – *If the velocity and temperature fields are even functions while the magnetic field is an odd function of the vertical coordinate z and only two terms of the secular equation are retained, then the approximate neutral curve reads*

$$R_0^e=\frac{(\pi^2+a^2)^3+M^2\pi^2(\pi^2+a^2)}{a^2},$$

if the Hall effect is absent and

$$R^e = \frac{A_2 H_2 + X_2^{(2)}}{a^2}, \tag{12}$$

or, equivalently,

$$R^e = \frac{A_1 H_1 + X_1^{(2)}}{a^2}, \tag{13}$$

if the Hall effect is present, where $X_2^{(2)}$ *has the expression* (11), $X_1^{(2)} = Q + X_2^{(2)}$. *In* $(a^2; R)$*-plane the curves* (13) *are situated below the curve* R_0^e, *thus the Hall effect is destabilizing.*

PROOF. – The equation in X_1 can be obtained from the equation in X_2 by simply replacing Q by $-Q$. Then it is immediate that $X_1^{(2)} < 0$, hence the values of R given by (13) are smaller than those given by R_0^e. Moreover, $X_1^{(1)} > 0$, hence the corresponding values for $R = [H_1 A_1 + X_1^{(1)}]a^{-2} = [H_2 A_2 + X_2^{(1)}]a^{-2}$ are higher than those given by R_0^e. Hence R_0^e is situated between the two curves from the case $\beta_H \neq 0$, but only the last one is of interest for us. Since $X_2^{(1),(2)} < 0$ it follows that both the corresponding curves $R = [H_2 A_2 + X_2^{(1),(2)}]a^{-2}$ are situated below the curve $R = A_2 H_2 a^{-2}$. Their expression computed by (11) show that as $\beta_H \to 0$ these curves tends to the curves $R = A_2 H_2 a^{-2}$ and $R = A_1 H_1 a^{-2}$ respectively from the case of the absence of the Hall effect. Therefore, for a sufficiently small β_H the lower curve is the closest to R_0^e. In addition, the neutral curve from the case $\beta_H = 0$ is situated between the two curves corresponding to the two solutions of the secular equation (10) with $\beta_H \neq 0$.

If more terms in (10) are retained then higher degree equations in X_i are obtained. Due to the decreasing order of magnitude [6] of the additional terms, their contribution to the solution diminishes and, thus, we expect that a limit neutral curve exist under that for the case $\beta_H = 0$. In fact, all involved series converge at least like n^{-1} as $n \to \infty$. In (10) the terms in Ra^2 are of order n^{-8} while those which do not contain Ra^2 are of order n^{-2} as $n \to \infty$ and they are negative. Indeed, for M^2, β_H and Ra^2 not too large, $X_n G_n + M^2 \beta_H^2 L_n^2 \sim X_n G_n \sim -A_n H_n^2 \sim -A_n^5$, $A_n X_n = A_n[Ra^2 - A_n H_n] = A_n Ra^2 - A_n^2 H_n$, therefore the coefficient of Ra^2 is of order A_n^{-4} and that of $A_n H_n$ is of order A_n^{-1} as $n \to \infty$. We emphasize that we are interested in the smallest R, therefore in the solution $X_k^{(k)}$ which corresponds to the value Ra^2 smaller than R_0^e. This solution exists. Indeed, all the smallest negative solutions of the k-th degree equation in X_k, obtained by truncating (10) to k terms, has a continuous dependence on β_H^2.

Moreover for $\beta_H^2 \to 0$ the corresponding Ra^2 tend to $H_1 A_1, H_2 A_2, \cdots, H_k A_k$. In particular, the lowest corresponding curve $R = R(a^2)$ will tend to the neutral curve defined by R_0^e.

5. – Neutral curves for the odd case.

Assume that the velocity and temperature are odd while the magnetic field is an even function of z. This case is referred to as the *odd case* and it corresponds to the problem (6″) (7″). Since, in this case the use of the total sets $\{E_1, E_3, \cdots\}$, $\{F_1, F_3, \cdots\}$ introduce four constraints, we use the total sets $\{1, E_2, E_4, \cdots\}$, $\{F_2, F_4, \cdots\}$. Then the Fourier coefficients of the unknown functions written as $f_o + \sum_{n=1}^{\infty} f_{2n} E_{2n}$ (for even functions f) and $\sum_{n=1}^{\infty} f_{2n} F_{2n}$ (for odd functions) satisfy the system

$$
(14) \quad \begin{cases}
-B_n K_{2n}^e + 2n\pi W_{2n}^o - \beta_H 2n\pi X_{2n}^o = 2\sqrt{2}(-1)^{n+1}[a_6 - \beta_H a_4], \\
-B_n Z_{2n}^e + M^2 2n\pi X_{2n}^o = 2\sqrt{2}(-1)^{n+1} a_4 M^2, \\
B_n^2 W_{2n}^o + M^2 B_n 2n\pi K_{2n}^e - \dfrac{P_m}{P_r} Ra^2 \Theta_{2n}^o = -2\sqrt{2}(-1)^{n+1} 2n\pi a_6 M^2, \\
-B_n X_{2n}^o - 2n\pi Z_{2n}^e + \beta_H 2n\pi B_n K_{2n}^o = -2\sqrt{2}(-1)^{n+1} 2n\pi [a_6 \beta_H + a_4], \\
-\dfrac{P_m}{P_r} B_n \Theta_{2n}^o + W_{2n}^o = 0, \\
-2a_6 + a^2 a_2 + 2\beta_H a_4 = 0, \\
-a^2 a_1 + 2M^2 a_4 = 0
\end{cases}
$$

and the constraints

$$
(15) \quad 2a_4 + \sum_{n=1}^{\infty} \Big[2\sqrt{2}(-1)^n a_4 + 2n\pi X_{2n}^o\Big](-1)^n \sqrt{2} = 0, \qquad a_2 + \sum_{n=1}^{\infty} (-1)^n \sqrt{2} K_{2n}^e = 0,
$$

where $a_4 = X^o(0.5)$, $X_o^{o(1)} = 2a_4$, $Z_o^e = a_1$, $K_o^e = a_2$, $DK^e(0.5) = a_6$, $K_o^{e(2)} = 2a_6$ and $B_n = (2n\pi)^2 + a^2$. From this system we obtain, for $n \geq 1$, $X_{2n}^o = \Delta_{4n}/\Delta_n$, $K_{2n}^e = \Delta_{2n}/\Delta_n$, where

$$
\Delta_n = \frac{P_m}{P_r} B_n \Big\{ (Ra^2 - B_n^3)\big[B_n^2 + (B_n - a^2)(M^2 + B_n \beta_H^2)\big]
$$

$$
-M^2 B_n (B_n - a^2)\big[B_n^2 + (B_n - a^2)M^2\big] \Big\},
$$

$$
\Delta_{4n} = 2\sqrt{2}(-1)^{n+1} \frac{P_m}{P_r} 2n\pi a_4 \Big[(Ra^2 - B_n^3)(B_n^2 + M^2 B_n + B_n^2 \beta_H^2)
$$

$$
-M^2 B_n^2 (B_n - a^2)(M^2 + B_n) \Big],
$$

$$
\Delta_{2n} = 2\sqrt{2}(-1)^{n+1} \frac{P_m}{P_r} \Big\{ a_4 a^2 \beta_H B_n (Ra^2 - B_n^3) - a_6 \frac{P_r}{P_m} \frac{\Delta_n}{B_n} \Big\}.
$$

Thus, the constraints imply the eigenvalue equation in the form of the following determinant containing infinite sums of series converging at least like n^{-1} as $n \to \infty$

$$\det \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0, \tag{16}$$

where

$$a_{11} = 1 + 2a^2 \sum_{n=1}^{\infty} B_n^2 \frac{[(Ra^2 - B_n^3) - M^2 B_n (B_n - a^2)]}{\Delta_n \frac{P_r}{P_m}}, \qquad a_{12} = 0,$$

$$a_{21} = \frac{-\beta_H}{a^2} - 2\sum_{n=1}^{\infty} \frac{a^2 \beta_H B_n (Ra^2 - B_n^3)}{\Delta_n \frac{P_r}{P_m}}, \qquad a_{22} = \frac{1}{a^2} + 2\sum_{n=1}^{\infty} \frac{1}{B_n}.$$

In the above we assumed that $\Delta_n \neq 0$.

5.1 – *The case* $\beta_H = 0$.

In this case $\Delta_n = \dfrac{P_m}{P_r} B_n[(Ra^2 - B_n^3) - M^2 B_n(B_n - a^2)][B_n^2 + M^2(B_n - a^2)]$ and $\Delta_{4n} = 2\sqrt{2}(-1)^{n+1} \dfrac{P_m}{P_r} 2n\pi a_4 B_n [(Ra^2 - B_n^3) - M^2 B_n (B_n - a^2)](B_n + M^2)$, $\Delta_{2n} = 2\sqrt{2}(-1)^n a_6 \frac{\Delta_n}{B_n}$, such that, if $\Delta_n \neq 0$, the constraints imply the secular equation

$$\left(1 + 2a^2 \sum_{n=1}^{\infty} \frac{B_n}{B_n^2 + M^2(B_n - a^2)}\right)\left(\frac{1}{a^2} + 2\sum_{n=1}^{\infty} \frac{1}{B_n}\right) = 0.$$

This relation cannot hold because all involved terms are positive. Hence, our problem has no eigenvalue. Then assume that $\Delta_n = 0$. In this case $Ra^2 = B_n^3 + M^2 B_n (B_n - a^2)$, or, with the notation $L_n = B_n(B_n - a^2)$ and $H_n = B_n^2 + M^2(B_n - a^2)$, we have the eigenvalues $R_n = \dfrac{H_n B_n}{a^2}$. Let us prove that, indeed, for these values non trivial solution of (6″), (7″) exist. First remark that for $\beta_H = 0$ system (6″) splits into two non coupled systems

$$\begin{cases} (D^2 - a^2) Z^e + M^2 D X^o = 0, \\ (D^2 - a^2) X^o + D Z^e = 0, \end{cases} \tag{17}$$

$$\begin{cases} (D^2 - a^2) K^e + D W^o = 0, \\ (D^2 - a^2)^2 W^o + M^2 D (D^2 - a^2) K^e - H_n B_n \dfrac{P_m}{P_r} \Theta^o = 0, \\ \dfrac{P_m}{P_r} (D^2 - a^2) \Theta^o + W^o = 0, \end{cases} \tag{18}$$

By eliminating Z^e between $D(17_1)$ and (17_2) it follows $(D^2-a^2)^2X^o - M^2DX^o = 0$, while taking into account in (17_2) the boundary conditions for Z^e it follows the supplementary boundary conditions $(D^2-a^2)X^o = 0$ at $z = \pm 0.5$. If $\lambda_1 \neq \lambda_2$ are the roots of the corresponding characteristic equation$(\lambda^2-a^2)^2 - M^2\lambda^2 = 0$, introducing the general odd solution $X^o = A\sinh\lambda_1 + B\sinh\lambda_2$ into these conditions, we obtain the secular equation

$$\frac{(\lambda_2^2-a^2)\tanh\frac{\lambda_2}{2}}{\lambda_2} = \frac{(\lambda_1^2-a^2)\tanh\frac{\lambda_1}{2}}{\lambda_1}.$$

The function $f(\lambda) = \lambda^{-1}(\lambda^2-a^2)\tanh\dfrac{\lambda}{2}$ is monotone for $\lambda > 0$ and for $\lambda < 0$. Indeed, $\dfrac{df}{d\lambda} = 0$ reads $\sinh\lambda = -\lambda(\lambda^2-a^2)(\lambda^2+a^2)^{-1}$ and the graphs of the functions defined by the two sides of this equality are intersecting only for $\lambda = 0$. Therefore the secular equation has only the trivial solution. Consequently X^o and Z^e are trivial functions.

Consider now the system (18) with the corresponding boundary conditions from $(7'')$. Then, using the expansion of the unknown functions on the total sets $\{E_{2n}\}_{n\in\boldsymbol{N}}$ and $\{F_{2n}\}_{n\in\boldsymbol{N}^*}$ we have

$$(19)\quad \begin{cases} -B_nK_{2n}^e + 2n\pi W_{2n}^o = 2\sqrt{2}(-1)^{n+1}a_6, \\ B_n^2W_{2n}^o + M^2B_n2n\pi K_{2n}^e - \dfrac{P_m}{P_r}B_nH_n\Theta_{2n}^o = -2\sqrt{2}(-1)^{n+1}2n\pi a_6M^2, \\ -\dfrac{P_m}{P_r}B_n\Theta_{2n}^o + W_{2n}^o = 0, \\ -2a_6 + a^2a_2 = 0. \end{cases}$$

Remark that $-2n\pi M^2(19)_1 + [B_n^2 + M^2(B_n - a^2)](19)_3 = (19)_2$, (we remind that $H_n = B_n^2 + M^2(B_n - a^2)$) therefore the equations $(19)_{1,2,3}$ are not independent, in other words, equations $(6'')_{1,3,5}$are not indipendent. This can be seen by performing $M^2D(18)_1 + [(D^2-a^2)^2 - M^2D^2](18)_3$ and add to $(18)_2$ to obtain

$$(20)\quad (D^2-a^2)^2W^o + M^2D(D^2-a^2)K^e + \frac{P_m}{P_r}[(D^2-a^2)^2 - M^2D^2](D^2-a^2)\Theta = 0.$$

On the other hand, the elimination of W^o and K^e between (18) leads to the following equation in Θ

$$(21)\quad [(D^2-a^2)^2 - M^2D^2](D^2-a^2)\Theta + Ra^2\Theta = 0,$$

therefore $\dfrac{P_m}{P_r}[(D^2-a^2)^2 - M^2D^2](D^2-a^2)\Theta = -\dfrac{P_m}{P_r}Ra^2\Theta$ and so, for $Ra^2 = B_nH_n$ (20) becomes $(18)_2$.

The operators M^2D and $(D^2 - a^2)^2 - M^2D^2$ were constructed by taking into account that a factor of $2n\pi i$ is generated by the application of the operator D and, so, B_n by $-(D^2 - a^2)$. This type of reasoning is generally useful when wishing to express properties of the system in Fourier coefficients in terms of those in the corresponding system of differential equations.

The characteristic equation for (21) reads

$$\left[(\lambda^2 - a^2)^2 - M^2\lambda^2\right](\lambda^2 - a^2) + Ra^2 = 0. \tag{22}$$

Therefore for every eigenvalue $R_n = B_nH_n/a^2 = a^{-2}[(2n\pi)^2 + a^2]\cdot\left\{[(2n\pi)^2 + a^2]^2 + M^2(2n\pi)^2\right\}$ we can find the six solutions of (22) such that the odd general solution Θ^o of (21) has the form $\Theta^o(z) = A_{1n}\sin(2n\pi z) + A_{3n}\sinh\lambda_3 z + A_{5n}\sinh\lambda_5 z$, where λ_3 and λ_5 are given in the following, Θ^o satisfies the boundary conditions given in (7″) or deduced from (6″) and (7″) and it is not identically equal to zero. The same can be said about W^o and K^e. Really, $R_n = H_nB_n/a^2$ reads, equivalently, as $R_n = B_nH_n/a^2 = a^{-2}[(2n\pi)^2 + a^2]\cdot\left\{[(2n\pi)^2 + a^2]^2 + M^2(2n\pi)^2\right\}$ and, thus, (22) can be written in the form

$$\lambda^6 - (3a^2 + M^2)\lambda^4 + a^2(3a^2 + M^2)\lambda^2 - a^6 + [(2n\pi)^2 + a^2]\left\{[(2n\pi)^2 + a^2]^2 + M^2(2n\pi)^2\right\} = 0. \tag{22'}$$

Since this equation has two roots $\lambda_{1,2} = \pm 2n\pi i$ it follows that the other four roots, written as $\lambda_{3,4} = \pm(\alpha + i\beta)$ and $\lambda_{5,6} = \pm(\alpha - i\beta)$ satisfy the equation

$$\lambda^4 - (3a^2 + M^2 + 4n^2\pi^2)\lambda^2 + (3a^2 + M^2)(a^2 + 4n^2\pi^2) + 16n^4\pi^4 = 0, \tag{22''}$$

whence the above quoted form for Θ^o. By construction this function satisfies the equation (21). It must satisfy also the following boundary conditions derived from (6″) and (7″)

$$\Theta^o = D^2\Theta^o = D^4\Theta^o = 0 \qquad z = \pm 0.5 \tag{23}$$

implying the secular equation in $\boldsymbol{C}$

$$\det\begin{vmatrix} \sin(n\pi) & \sinh\left(\frac{\lambda_3}{2}\right) & \sinh\left(\frac{\lambda_5}{2}\right) \\ -4n^2\pi^2\sin(n\pi) & \lambda_3^2\sinh\left(\frac{\lambda_3}{2}\right) & \lambda_5^2\sinh\left(\frac{\lambda_5}{2}\right) \\ 16n^4\pi^4\sin(n\pi) & \lambda_3^4\sinh\left(\frac{\lambda_3}{2}\right) & \lambda_5^4\sinh\left(\frac{\lambda_5}{2}\right) \end{vmatrix} = 0, \tag{24}$$

which is automatically satisfied for both real and complex values of λ_3 and λ_5. This determinant vanishes always. However, it is easy to check that there exists a 2×2 non vanishing minor formed with the minor of the upper-right corner of (24). The corresponding equations in A_{1n}, A_{3n}, A_{5n} read

$$A_{1n}\sin(n\pi)+A_{3n}\sinh(\alpha/2)\cos(\beta/2)+A_{5n}\sin(\beta/2)\cosh(\alpha/2)=0,$$

$$-4n^2\pi^2\sin(n\pi)A_{1n}+A_{3n}[(\alpha^2-\beta^2)\sinh(\alpha/2)\cos(\beta/2)-2\alpha\beta\sin(\beta/2)\cosh(\alpha/2)]+$$

$$A_{5n}[\sinh(\alpha/2)\cos(\beta/2)+(\alpha^2-\beta^2)\sin(\beta/2)\cosh(\alpha/2)]=0,$$

implying $A_{3n}=A_{5n}=0$. Therefore $\Theta^o=A_{1n}\sin(2n\pi z)$, where A_{1n} are determined up to a constant factor: they are the Fourier coefficients corresponding to the expansion functions $\sin(2n\pi z)$. Consequently for every R_n we have one non vanishing solution Θ^o of the above form, i.e.R_n is an eigenvalue, indeed. The system (19) gives the same result: $R_m=H_mB_m/a^2$ represent the eigenvalues for the problem (19), (7″). Indeed, since the Cramer determinant for $(19)_{1,2,3}$ is null, we choose equations $(19)_{1,3}$ in W^o_{2n} and K^e_{2n}. The corresponding Cramer determinant, which is a 2×2 minor of that for $(19)_{1,2,3}$, is non vanishing. Therefore we can determine uniquely W^o_{2n} and K^e_{2n} in terms of Θ^o_{2n} and a_6, i.e. $W^o_{2m}=\dfrac{P_m}{P_r}B_m\Theta^o_{2m}$ and $K^e_{2m}=2m\pi\Theta^o_{2m}\dfrac{P_m}{P_r}+\dfrac{2\sqrt{2}(-1)^m a_6}{B_m}$, while for $m\neq n$ we have $W^o_{2n}=\Theta^o_{2n}=0$, $K^e_{2n}=\dfrac{2\sqrt{2}(-1)^n a_6}{B_n}$. Since, by $(19)_4$, $a_2=2a_6a^{-2}$, the constraint $(15)_2$ becomes $\dfrac{2a_6}{a^2}+\sqrt{2}(-1)^m 2m\pi\Theta^o_{2m}\dfrac{P_m}{P_r}+\sum_{n=1}^{\infty}4\dfrac{a_6}{B_n}=0$, or, because $\sum_{n=1}^{\infty}\dfrac{1}{4n^2\pi^2+a^2}=\dfrac{1}{4a}\coth\dfrac{a}{2}-\dfrac{1}{2a^2}$, it follows that $a_6=(-1)^{m+1}\cdot\sqrt{2}a\tanh\dfrac{a}{2}2m\pi\dfrac{P_m}{P_r}\Theta^o_{2m}$.

Taking into account formula [10] $\sum_{k=1}^{\infty}\dfrac{(-1)^k\cos kx}{k^2+b^2}=\dfrac{\pi}{2b}\dfrac{\cosh bx}{\sinh b\pi}-\dfrac{1}{2b^2}$, $-\pi\le x\le\pi$, we find

$$K^e(z)=2\sqrt{2}m\pi\frac{P_m}{P_r}\Theta^o_{2m}\Big[(-1)^{m+1}\frac{\cosh az}{\cosh a/2}+\cos(2m\pi z)\Big].$$

In addition $W^o(z)=\sqrt{2}\dfrac{P_m}{P_r}B_m\Theta^o_{2m}\sin(2m\pi z)$ and $\Theta^o(z)=\Theta^o_{2m}\sqrt{2}\sin(2m\pi z)$, hence $R_m=B_mH_ma^{-2}$ is an eigenvalue. The neutral curve corresponds to $m=1$ i.e. it has the equation

$$R=\frac{(4\pi^2+a^2)^3+4M^2\pi^2(4\pi^2+a^2)}{a^2}. \tag{25}$$

Let us prove that no $R_n\neq B_nH_na^{-2}$, $n\in\boldsymbol{N}^*$, is an eigenvalue. Indeed (19) has only trivial solutions. In this case the Cramer determinant is $\Delta''_n=\dfrac{P_m}{P_r}B_n(Ra^2-B_nH_n)$ and $W^o_{2n}=\Theta^o_{2n}=0$, $K^e_{2n}=\dfrac{2\sqrt{2}(-1)^n a_6}{B_n}$, which introduced into the constraint implies $a_6 = 0$, hence $K^e_{2n} = 0 = a_2$, hence the desired result.

5.2 – *The case* $\beta_H \neq 0$.

In this case the secular equation (16) becomes

$$1 + 2a^2 \sum_{n=1}^{\infty} \frac{(Ra^2 - B_n H^o_n) B_n}{(Ra^2 - B^3_n)(H^o_n + \beta^2_H L^o_n) - M^2 L^o_n H^o_n} = 0 \tag{26}$$

or, equivalently

$$1 + 2a^2 \sum_{n=1}^{\infty} \frac{(Ra^2 - B_n H^o_n) B_n}{(Ra^2 - B_n H^o_n) G^o_n + \beta^2_H M^2 L^{o^2}_n} = 0 \tag{26'}$$

where $L^o_n = B_n(B_n - a^2)$, $H^o_n = B^2_n + M^2(B_n - a^2)$ and $G^o_n = H^o_n + \beta^2_H L^o_n$.

If in the sums in (26′) a single terms is retained we obtain

$$R = \frac{(4\pi^2 + a^2)^3 + 4M^2\pi^2(4\pi^2 + a^2)}{a^2} - \beta^2_H \frac{16M^2\pi^4(4\pi^2 + a^2)^2}{a^2\{(4\pi^2 + a^2)^2 + 4M^2\pi^2 + 2a^2(4\pi^2 + a^2) + \beta^2_H 4\pi^2(4\pi^2 + a^2)\}} \tag{27}$$

showing the instabilizing effect of the Hall current if compared with (25).

Now let truncate (26′) up to terms corresponding to $n = 2$ and introduce the notation $V^o_n = (Ra^2 - B_n H^o_n) M^{-2} L^{o^{-1}}_n$. Then $V^o_2 = \dfrac{V^o_1 M^2 L_1 - Q^o}{M^2 L_2}$ and so, (26′) becomes

$$(V^o_1 - \overline{V}^o_1)^2 M^2 L_1 T^2_1 (N^o_2 T_1 + 2a^2 B_2 N^o_1) + (V^o_1 - \overline{V}^o_1) T_1 [M^2 L_2 P_2 T^2_1 - (P^o_1 M^2 L^o_1 + Q^o T_1) (N^o_2 T_1 + 2a^2 B_2 N^o_1) + 4a^4 B_1 B_2 M^2 L^o_1 P^o_1] - 4a^4 B_1 B_2 P^o_1 (P_1 M^2 L^o_1 + Q^o T_1) = 0 \tag{28}$$

where $N^o_n = H_n + \beta^2_H L^o_n$, $P^o_n = \beta^2_H L^o_n$, $T_1 = N^o_1 + 2a^2 B_1$, $Q^o = H^o_2 B_2 - H^o_1 B_1$ and $\overline{V}^o_1$ is the solution of (26′) corresponding to $n = 1$, i.e. $\overline{V}^o_1 = -P_1 T^{-1}_1$. Due to the fact that $Ra^2 - B_1 H^o_1 = V^o_1 M^2 L^o_1 = \overline{V}^o_1 M^2 L^o_1 + (V^o_1 - \overline{V}^o_1) M^2 L^2_1$ this means that the case with two terms shows destabilizing effect if $V^o_1 - \overline{V}^o_1 < 0$ and stabilizing one otherwise. The equation (28) shows that we have one positive solution $V^o_1 - \overline{V}^o_1$ and other negative. It follows that the neutral curve corresponding to this negative solution is better than (27).

6. – Conclusions.

In order to obtain neutral curves for the Bénard magnetic convection, in the presence of the Hall currents and for free boundaries, the Budiansky -DiPrima method was used. It involves Fourier series expansion of the normal modes on total sets of very simple functions (cosine and sine), in associate separable Sobolev spaces, and satisfying part of the boundary conditions of the problem. The other boundary conditions introduce some constraints. The system of ordinary differential equations was split into two systems for even velocity and temperature fields (*the even case*) and conversely (*the odd case*). In *the even case* we used total sets leading to two constraints while in *the odd case* the same choice would imply four constraints.This is why in the odd case we used other total sets leading also to two constraints.The secular equation defining the neutral curves were obtained in the form of a vanishing determinant the entries of which were series involving the Fourier coefficients of the unknown functions. Retaining one and two terms in these series we determined the closed form solution of the equation defining the neutral curve that for two terms was situated under the neutral curve corresponding to one term. This was to be expected because the involved series were convergent at least as n^{-2} as $n \to \infty$. In the first *even case* it was found that if the sums were truncated to a single term, the stability bounds corresponds to the case $\beta_H = 0$. In the case ot two terms the neutral curves for $\beta_H \neq 0$ were situated below the neutral curves from the case $\beta_H = 0$. In the *odd case* the Hall effect was present even for a single term in the sums. When trying to compare our results with the case when the Hall effect is absent we found that our calculations implied a singularity and so we treated this case separately. The comparison between the case of absence and presence of the Hall current showed a destabilizing effect of this one.

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