Bollettino
Unione Matematica Italiana

Massoud Amini

Fourier Transform of Unbounded Measures on Hypergroups

Bollettino dell'Unione Matematica Italiana, Serie 8, Vol. **10-B** (2007), n.3, p. 819–828.

Unione Matematica Italiana

<http://www.bdim.eu/item?id=BUMI_2007_8_10B_3_819_0>

L'utilizzo e la stampa di questo documento digitale è consentito liberamente per motivi di ricerca e studio. Non è consentito l'utilizzo dello stesso per motivi commerciali. Tutte le copie di questo documento devono riportare questo avvertimento.

Articolo digitalizzato nel quadro del programma
bdim (Biblioteca Digitale Italiana di Matematica)
SIMAI & UMI
http://www.bdim.eu/

Bollettino dell'Unione Matematica Italiana, Unione Matematica Italiana, 2007.

Bollettino U. M. I.
(8) **10**-B (2007), 819-828

Fourier Transform of Unbounded Measures on Hypergroups

MASSOUD AMINI

Sunto. – *Proviamo che una misura illimitata su un ipergruppo fortemente commutativo è trasformabile se e solo se la sua convoluzione con una qualunque funzione definita positiva a supporto compatto è definita positiva.*

Summary. – *We show that an unbounded measure on a strong commutative hypergroup is transformable if and only if its convolution with any positive definite function of compact support is positive definite.*

1. – Introduction.

Fourier and Fourier-Stieltjes transform play a central role in the theory of absolutely integrable functions and bounded Borel measures on a locally compact abelian group G [R]. They are particularly important because they map the group algebra $L^1(G)$ and measure algebra $M(G)$ onto the Fourier algebra $A(G)$ and Fourier-Stieltjes algebra $B(G)$, respectively. There are important Borel measures on a locally compact, non-compact, abelian group G which are unbounded. A typical example is a left Haar measure. L. Argabright and J. de Lamadrid in [AL] explored a generalized Fourier transform of unbounded measures on locally compact abelian groups. This theory has recently been successfully applied to the study of quasi-crystals [B].

A version of generalized Fourier transform is defined for a class of commutative hypergroups [BH]. Some of the main results [AL] are stated and proved in [BH] for hypergroups, but the important connection with Fourier and Fourier-Stieltjes spaces are not investigated in [BH] (in contrast with the group case, these may fail to be closed under pointwise multiplication for general hypergroups.) Recently these spaces are studied in [AM], and under some conditions, they are shown to be Banach algebras. In this paper we investigate the relation between transformablity of unbounded measures on strong commutative hypergroups and these spaces. In the next section we study unbounded measures on locally compact (not necessarily commutative) hypergroups. The main objective of this section is the study of translation bounded measures. These are studied in [BH] in commutative case. The main results of this paper (Theorem 3.8 and Corollary 3.10) are stated and proved in section 3. The former states that an unbounded measure on a strong commutative hypergroup is transformable if and only if its convolution with any positive definite function of compact support is positive definite. The latter gives the condition for the transform of this measure to be a function.

2. – Unbounded measures.

Let K be a locally compact hypergroup (a convo in the sense of [Je]). We denote the spaces of bounded continuous functions and continuous functions of compact support on K by $C_b(K)$ and $C_c(K)$, respectively. The latter is an inductive limit of Banach spaces $C_A(K)$ consisting of functions with support in A, where A runs over all compact subsets of K. This in particular implies that if X is a Banach space, a linear transformation $T : C_c(K) \to X$ is continuous if and only if it locally bounded, that is for each compact subset $A \subseteq K$ there is $\beta = \beta_A > 0$ such that

$$\|T(f)\| \leq \beta \|f\|_\infty \quad (f \in C_A(K)).$$

Also a version of closed graph theorem is valid for $C_c(K)$, namely T is locally bounded if and only if it has a closed graph [Bo]. All over this paper $C_c(K)$ is considered with the inductive limit topology (*ind*). Following [Bo] we have the following definitions.

DEFINITION 2.1. – *A measure on K is an element of $C_c(K)^*$. The space of all measures on K is denoted by $M(K)$. The subspaces of bounded and compactly supported measures are denoted by $M_b(K)$ and $M_c(K)$, respectively.*

DEFINITION 2.2. – *For $\mu, \upsilon \in M(K)$, we say that μ is convolvable with υ if for each $f \in C_c(K)$, the map $(x, y) \mapsto f(x * y)$ is integrable over $K \times K$ with respect to the product measure $|\mu| \times |\upsilon|$. In this case $\mu * \upsilon$ is defined by*

$$\int_K f d(\mu * \upsilon) = \int_K \int_K f(x * y) d\mu(x) d\upsilon(y) = \int_K \int_K \int_K f(t) d(\delta_x * \delta_y)(t) d\mu(x) d\upsilon(y).$$

Let $C(\upsilon)$ denote the set of all measures convolvable with υ. When K is a measured hypergroup with a left Haar measure m, a locally integrable function f is called convolvable with υ if $fm \in C(\upsilon)$. In this case, we put $f * \upsilon = fm * \upsilon$. Similarly if $\upsilon \in C(fm)$ then $\upsilon * f = \upsilon * fm$. The next two lemmas are a straightforward calculation and we omit the proof. Here Δ denotes the modular function of K. Also for a function f on K, we define $\bar{f}$ and $\tilde{f}$ by

$$\bar{f}(x) = f(\bar{x}), \tilde{f}(x) = \overline{f(\bar{x})} \quad (x \in K).$$

We denote the complex conjugate of f by f^-. For $\mu \in M(K)$, μ^-, $\bar{\mu}$ and $\tilde{\mu}$ are defined similarly.

LEMMA 2.3. – *If K is a measured hypergroup, $\mu \in M(K)$, and f is locally integrable on K, then*

(i) $(\mu * f)(x) = \int_K f(\bar{y} * x) d\mu(y) = \int_K f d(\bar{\mu} * \delta_x)$,

(ii) $(f * \mu)(x) = \int_K f(x * \bar{y}) \bar{\Delta}(y) d\mu(y) = \int_K f d(\delta_x * \Delta\bar{\mu})$,

locally almost everywhere. If $f \in C_c(K)$, the above formulas are valid everywhere and define continuous (not necessarily bounded) functions on K.

If we want to avoid the modular function in the second formula, we should define the convolution of functions and measures differently, but then this definition does not completely match with the formula for convolution of measures (see section of [BH]).

LEMMA 2.4. – *If K is a measured hypergroup, $\mu, \upsilon \in M(K)$, $f \in C_c(K)$, and $\mu \in C(\upsilon)$, then $\bar{\mu} \in C(f)$, $\bar{\mu} * f \in L^1(\upsilon)$, and*

$$\int_K f d\mu * \upsilon = \int_K \bar{\mu} * f d\upsilon.$$

DEFINITION 2.5. – *A measure $\mu \in M(K)$ is called left translation bounded if for each compact subset $A \subseteq K$*

$$\ell_\mu(A) := \sup_{x \in K} |\bar{\mu} * \delta_x|(A) < \infty.$$

Similarly μ is called right translation bounded if for each compact subset $A \subseteq K$

$$r_\mu(A) := \sup_{x \in K} |\delta_x * \Delta\bar{\mu}|(A) < \infty.$$

We denote the set of left and right translation bounded measures on K by $M_{\ell b}(K)$ and $M_{rb}(K)$, respectively.

PROPOSITION 2.6. – *Let $\mu \in M(K)$, then*

(i) μ is left translation bounded if and only if $\mu * f \in C_b(K)$, for each $f \in C_c(K)$. In this case for each compact subset $A \subseteq K$,

$$\|\mu * f\|_\infty \leq \ell_\mu(A) \|f\|_\infty \quad (f \in C_A(K)).$$

(ii) μ is right translation bounded if and only if $f * \mu \in C_b(K)$, for each $f \in C_c(K)$. In this case for each compact subset $A \subseteq K$,

$$\|f * \mu\|_\infty \leq r_\mu(A)\|f\|_\infty \quad (f \in C_A(K)).$$

PROOF. – We prove (*i*), (*ii*) is proved similarly. If $f \in C_A(K)$, then by Lemma 2.1,

$$|(\mu * f)(x)| \leq \|f\|_\infty |\bar{\mu} * \delta_x|(A) \leq \|f\|_\infty \ell_\mu(A),$$

for each $x \in K$. Conversely, if $\mu * f$ is bounded for each $f \in C_c(K)$, then $f \mapsto \mu * f$ defines a linear map from $C_c(K)$ into $C_b(K)$. We claim that it has a closed graph. If $f_a \to 0$ in (*ind*), then there is a compact subset $A \subseteq K$ such that eventually $supp(f_a) \subseteq A$ and $f_a \to 0$, uniformly on A. If $\mu * f_a \to g$, uniformly on K, then $|(\mu * f_a)(x)| \leq \|f\|_\infty |\bar{\mu} * \delta_x|(A) \to 0$, for each $x \in K$. Hence $g = 0$. By closed graph theorem, for each compact subset $B \subseteq K$, there is $k_B > 0$ such that

$$\|\mu * f\|_\infty \leq k_B \|f\|_\infty \quad (f \in C_B(K)).$$

Now let $A \subseteq K$ be compact and choose $B \subseteq K$ compact with $B^\circ \supseteq A$. By Lemmas 2.1 and 2.2, for each $x \in K$,

$$\begin{aligned} |\bar{\mu} * \delta_x|(A) &= \sup\left\{\left|\int_K g d(\bar{\mu} * \delta_x)\right| : \|g\|_\infty \leq 1, supp(g) \subseteq A\right\} \\ &\leq \sup\left\{\left|\int_K g d(\bar{\mu} * \delta_x)\right| : \|g\|_\infty \leq 1, supp(g) \subseteq B\right\} \\ &= \sup\{|\mu * g(x)| : \|g\|_\infty \leq 1, supp(g) \subseteq B\} \leq k_B. \end{aligned}$$

□

EXAMPLE 2.7. – All elements of $M_b(K)$ and $L^p(K, m)$ are translation bounded. Also each hypergroup has a left-translation invariant measure n [Je, 4.3C]. We have

$$(n * f)(x) = \int_K (\delta_x * \bar{f}) dn \leq \int_K \bar{f} dn,$$

for each $x \in K$ and $f \in C_c^+(K)$. Hence $n \in M_{\ell b}(K)$. Similarly $\Delta n \in M_{rb}(K)$.

LEMMA 2.8. – *(i) If $\theta \in M_c(K)$ then $\theta \in C(\mu)$ and $\mu \in C(\theta)$, for each $\mu \in M(K)$.*

(ii) If $\upsilon \in M(K)$, then

a) $\upsilon \in M_{\ell b}(K)$ if and only if $\mu \in C(\upsilon)$, for each $\mu \in M_b(K)$.

b) $\upsilon \in M_{rb}(K)$ if and only if $\upsilon \in C(\mu)$, for each $\mu \in M_b(K)$.

PROOF. – (i) follows from the fact that $|\theta|^{\sim} * |f| \in C_c(K)$ for each $f \in C_c(K)$ [Je, 4.2F]. If $f \in C_c(K)$, $\upsilon \in M_{\ell b}(K)$ and $\mu \in M_b(K)$, then $|\upsilon| * |\bar{f}| \in C_b(K)$, hence

$$\int_K \int_K |f(x * y)| d|\mu|(x) d|\upsilon|(y) = \int_K (|\upsilon| * |\bar{f}|) d|\mu| < \infty,$$

so $\mu \in C(\upsilon)$. Conversely, if $\upsilon \notin M_{\ell b}(K)$ then there is $f \in C_c^+(K)$ such that $\upsilon * f$ is unbounded. Hence there is $\mu \in M_b^+(K)$ with $\upsilon * f \notin L^1(K)$, i.e.

$$\int_K \bar{f}(x * y) d\bar{m}(x) d\upsilon(y) = \int_K (\upsilon * f) d\mu = \infty,$$

that is $\bar{\mu} \notin C(\upsilon)$. This proves (iia), (iib) is similar. □

COROLLARY 2.9. – *If $\upsilon \in M_{\ell b}(K)$ then for each $\mu \in M_b(K)$ and $f \in C_c(K)$ we have $\mu * f \in L^1(K, \upsilon)$ and*

$$\int_K (\mu * f) d\upsilon = \int_K (\upsilon * \bar{f}) d\mu.$$

Following [Bo, chap. 8], we have the following associativity result which follows from the above lemma and a straightforward application of Fubini's Theorem.

THEOREM 2.10. – (i) *If $\mu, \upsilon \in M(K)$, $\mu \in C(\upsilon)$, and $\theta \in M_c(K)$, then $\theta \in C(\mu) \cap C(\mu * \upsilon)$ and $\theta * \mu \in C(\upsilon)$, and*

$$(\theta * \mu) * \upsilon = \theta * (\mu * \upsilon).$$

(ii) *If $\upsilon \in M_{\ell b}(K)$, $\mu_1, \mu_2 \in M_b(K)$ then $\mu_1 * (\mu_2 * \upsilon) = (\mu_1 * \mu_2) * \upsilon$.*

3. – Transformable measures.

In this section we assume that K is a commutative hypergroup such that $\hat{K}$ is a hypergroup, namely K is strong [BH, 2.4.1]. We don't assume that $(\hat{K})\hat{} = K$, unless otherwise specified. We denote the Haar measure on K and $\hat{K}$ by m_K and $m_{\hat{K}}$, respectively. For $\mu \in M_b(K)$, $\hat{\mu} \in C_b(\hat{K})$ is defined by

$$\hat{\mu}(\gamma) = \int_K \overline{\gamma(x)} d\mu(x) \quad (\gamma \in \hat{K}).$$

We usually identify $\hat{\mu}$ with $\hat{\mu} m_{\hat{K}}$. Put $\check{\mu} = (\bar{\hat{\mu}})$.

DEFINITION 3.1 (BH, 2.3.10). – *A measure* $\mu \in M(K)$ *is called transformable if there is* $\hat{\mu} \in M^+(\hat{K})$ *such that*

$$\int_K (f * \tilde{f}) d\mu = \int_{\hat{K}} |\check{f}| d\hat{\mu} \quad (f \in C_c(K)).$$

We denote the space of transformable measures on K by $M_t(K)$ and put $\hat{M}_t(\hat{K}) = \{\hat{\mu} : \mu \in M_t(K)\}$. Also we put $C_2(K) = span\{f * \tilde{f} : f \in C_c(K)\}$. Then the above relation could be rewritten as

$$\int_K g d\mu = \int_{\hat{K}} \check{g} d\hat{\mu} \quad (g \in C_2(K)),$$

or

$$\int_K f * g d\mu = \int_{\hat{K}} \check{f} \check{g} d\hat{\mu} \quad (f, g \in C_c(K)).$$

EXAMPLE 3.2. – The Haar measure m_K is transformable and $\hat{m}_K = \pi_K$ is the Levitan-Plancherel measure on $\hat{K}$ [BH, 2.2.13]. Also all bounded measures are transformable. If $\mu \in M_b(K)$ and $\hat{\mu} \geq 0$ on $supp(\pi_K)$, then the transform of μ in the above sense is $\hat{\mu}\pi_K$, where $\hat{\mu}$ is the Fourier-Stieltjes transform of μ. In particular, $\hat{\delta}_e = \pi_K$ [BH]. Finally, if f is a bounded positive definite function on K, then by Bochner's Theorem [BH, 4.1.16], there is a unique $\sigma \in M_b^+(K)$ such that $f = \check{\sigma}$. Then it is easy to see that $fm_K \in M_t(K)$ and $(fm_K)\hat{} = \tilde{\sigma}$.

LEMMA 3.3. – $C_2(K)$ *is dense in* $C_c(K)$.

PROOF. – Let $\{u_a\}$ be a bounded approximate identity of $L^1(K, m_K)$ consisting of elements of $C_c(K)$ with $u_a \geq 0$, $u_a = \bar{u}_a$, and $\int_K u_a dm_K = 1$ such that $supp(u_a)$ is contained in a compact neighborhood V of e [MG]. For $f \in C_c(K)$ with $supp(f) = W$, we have $supp(f), supp(f * u_a) \subseteq W * V =: U$. By uniform continuity of f, given $\varepsilon > 0$, there is a_0 such that $|f(x * y) - f(x)| < \varepsilon$, for each $x \in U$ and $y \in supp(u_a)$ with $a \geq a_0$.

$$|(f * u_a - f)(x)| \leq \int_K |f(x * y) - f(x)| |u_a(y)| dm_K(y) = \int_V \varepsilon u_a dm_K = \varepsilon,$$

for $a \geq a_0$. □

By the above lemma and an argument similar to [AL, Thm. 2.1] we have

THEOREM 3.4 (Uniqueness Theorem). – *If* $\mu \in M_t(K)$ *then* μ *and* $\hat{\mu}$ *determine each other uniquely and the map* $\mu \mapsto \hat{\mu}$ *is an isomorphism of* $M_t(K)$ *onto* $\hat{M}_t(\hat{K})$.

The proof of the next result is straightforward.

LEMMA 3.5. – *For each* $\mu \in M_t(K)$, *the following measure are transformable with the given transform.*

(*i*) $\hat{\bar{\mu}} = \bar{\hat{\mu}}$,
(*ii*) $(\tilde{\mu})\hat{} = (\hat{\mu})^{-}$,
(*iii*) $(\mu^{-})\hat{} = (\hat{\mu})\tilde{}$,
(*iv*) $(\delta_x * \mu)\hat{} = \bar{x}\hat{\mu} \quad (x \in K)$,
(*v*) $(\gamma\mu)\hat{} = \delta_{\bar{\gamma}} * \hat{\mu} \quad (\gamma \in \hat{K})$.

If $\mu \in M_t(K)$, then by definition we have

$$\hat{C}_2(\hat{K}) := \{\hat{g} : g \in C_2(K)\} \subseteq L^2(\hat{K}, \hat{\mu}).$$

Next lemma is proved with the same argument as in [AL, Prop. 2.2]. We bring the proof for the sake of completeness.

LEMMA 3.6. – $\hat{C}_2(\hat{K}) \subseteq L^2(\hat{K}, \hat{\mu})$ *is dense.*

PROOF. – Since $C_c(\hat{K}) \subseteq L^2(\hat{K}, \hat{\mu})$ is dense, we need to show that given $\varphi \in C_c(\hat{K})$ and $\varepsilon > 0$, there is $g \in C_2(K)$ such that $\int_{\hat{K}} |\varphi - \hat{g}|^2 d\hat{\mu} < \varepsilon^2$.

Put $A = supp(\varphi)$. If $|\hat{\mu}|(A) = 0$, we take $g = 0$. Assume that $|\hat{\mu}|(A) > 0$. By [BH, 2.2.4(iv)], $\hat{C}_c(\hat{K})$ is dense in $C_0(\hat{K})$, so there is $f \in C_c(K)$ such that

$$|\hat{f}(\gamma) - 1| < \varepsilon \|\varphi\|_\infty^{-1} (|\hat{\mu}|(A))^{\frac{-1}{2}} =: \delta \quad (\gamma \in A).$$

We may assume that $\delta < \dfrac{1}{2}$, so $|\hat{f}| > \dfrac{1}{2}$, and so $\int_{\hat{K}} |\hat{f}|^2 d\hat{\mu} \neq 0$. Choose $h \in C_c(K)$ such that $\|\hat{h} - \varphi\|_\infty < \varepsilon (\int_{\hat{K}} |\hat{f}|^2 d\hat{\mu})^{\frac{-1}{2}}$. ut $g = h * f \in C_2(K)$, then

$$\left(\int_{\hat{K}} |\hat{g} - \varphi|^2 d\hat{\mu}\right)^{\frac{1}{2}} = \left(\int_{\hat{K}} |\hat{h}\hat{f} - \varphi|^2 d\hat{\mu}\right)^{\frac{1}{2}}$$
$$\leq \left(\int_{\hat{K}} |\hat{h}\hat{f} - \varphi\hat{f}|^2 d\hat{\mu}\right)^{\frac{1}{2}} + \left(\int_{\hat{K}} |\varphi\hat{f} - \varphi|^2 d\hat{\mu}\right)^{\frac{1}{2}} < 2\varepsilon. \qquad \square$$

LEMMA 3.7. – *If* $\mu \in M_t(K)$ *and* $g \in C_2(K)$, *then* $\hat{g} \in L^1(\hat{K}, \hat{\mu})$ *and* $g * \mu = (\hat{g}\hat{\mu})\check{}$.

PROOF. – The first assertion follows from definition of $\hat{\mu}$. For the second, since K is unimodular, given $x \in K$,

$$(g * \mu)(x) = \int_K g d(\delta_x * \bar{\mu}) = \int_{\hat{K}} \check{g}(\gamma)\gamma(\bar{x}) d\hat{\bar{\mu}}$$

$$= \int_{\hat{K}} \hat{g}(\gamma)\gamma(x) d\hat{\mu} = (\hat{g}\hat{\mu})\check{}. \qquad \square$$

Now we are ready to prove the main result of this paper.

THEOREM 3.8. – *For $\mu \in M(K)$, the following are equivalent:*

(*i*) $\mu \in M_t(K)$,
(*ii*) $g * \mu \in B(K)$, *for each* $g \in C_2(K)$.

PROOF. – If $g \in C_2(K)$, then $\hat{g} \in L^1(\hat{K}, \hat{\mu})$, hence $\hat{g}\hat{\mu} \in M_b(\hat{K})$. Therefore, by the above lemma and [BH, 4.1.15], $g * \mu = (\hat{g}\hat{\mu})\check{} \in B(K)$.

Conversely if $g * \mu \in B(K)$, for each $g \in C_2(K)$, then by Bochner's Theorem, there is a unique $\upsilon_g \in M_b(\hat{K})$ such that

$$g * \mu(x) = \int_{\hat{K}} \gamma(x) d\upsilon_g(\gamma) \quad (x \in K).$$

For each $f \in L^1(K, m)$,

$$\int_K f(g * \mu) dm = \int_K \int_{\hat{K}} f(x)\gamma(x) d\upsilon_g(\gamma) dm(x) = \int_{\hat{K}} \check{f} d\upsilon_g.$$

Fix $g, h \in C_2(K)$, then for each $f \in C_c(K)$,

$$\int_K (f * \bar{g})(h * \mu) dm = \int_K (f * \bar{g} * \bar{h}) d\mu = \int_K (f * \bar{h} * \bar{g}) d\mu = \int_K (f * \bar{h})(g * \mu) dm.$$

Hence

$$\int_{\hat{K}} \check{f}\hat{g} d\upsilon_h = \int_{\hat{K}} \check{f}\hat{h} d\upsilon_g.$$

By density of $\hat{C}_c(\hat{K})$ in $C_0(\hat{K})$, we get $\hat{g} d\upsilon_h = \hat{h} d\upsilon_g$. Define $\hat{\mu}$ on $\hat{K}$ by

$$\int_{\hat{K}} \psi d\hat{\mu} = \int_{\hat{K}} \frac{\psi}{\hat{h}} d\hat{\upsilon}_h,$$

where $h \in C_2(K)$ is such that $\hat{h} > 0$ on $supp(\psi)$. This is a well defined linear functional by what we just observed. It is easy to see that this is locally bounded on $C_c(\hat{K})$. Also

$$\int_{\hat{K}} \psi\hat{g}d\hat{\mu} = \int_{\hat{K}} \frac{\psi\hat{g}}{\hat{h}} d\upsilon_h = \int_{\hat{K}} \psi d\upsilon_g,$$

that is $\hat{g}d\hat{\mu} = d\upsilon_g$, and so $\hat{g} \in L^1(\hat{K}, \hat{\mu})$ and

$$\int_K g d\mu = (\bar{g} * \mu)(e) = \int_{\hat{K}} d\upsilon_{\hat{g}} = \int_{\hat{K}} \check{g} d\hat{\mu}.$$

Finally, since $\hat{h} > 0$ on $supp(\psi)$, we have $\int_{\hat{K}} \psi d\hat{\mu} \geq 0$, for $\psi \geq 0$. These all together show that $\hat{\mu} \in M^+(\hat{K})$ is the transform of μ and we are done. □

In [AM] the authors introduced the concept of tensor hypergroups and showed that for a tensor hypergroup K, the Fourier space $A(K)$ is a Banach algebra. The following lemma follows from Plancherel Theorem exactly as in the group case [E, 3.6.2°].

LEMMA 3.9. – *If K is a commutative, strong, tensor hypergroup, then $A(K)$ is isometrically isomorphic (through Fourier transform) to $L^1(\hat{K})$.*

The last result of this paper is a direct consequence of the above lemma and Theorem 3.8 (see the proof of [AL, Theorem 2.4]).

COROLLARY 3.10. – *If K is a commutative, strong, tensor hypergroup, then for each $\mu \in M(K)$, the following are equivalent:*

(i) $\mu \in M_t(K)$ *and* $\hat{\mu}$ *is a function,*
(ii) $g * \mu \in A(K)$, *for each* $g \in C_2(K)$.

REFERENCES

[AL] L. ARGABRIGHT - J. DE LAMADRID, *Fourier analysis of unbounded measures on locally compact abelian groups*, Memoirs of the American Mathematical Society, No. 145, American Mathematical Society, Providence, R.I., 1974.
[AM] M. AMINI - A. MEDGHALCHI, *Fourier algebras on tensor hypergrups*, Contemporary Math. **363** (2004), 1-14.
[B] M. BAAKE, *Diffraction of weighted lattice subsets*, Canad. Math. Bull. **45**, no. 4 (2002), 483-498.
[BH] W. R. BLOOM, H. HEYER, *Harmonic analysis of probability measures on hypergroups, de Gruyter Studies in Mathematics 20*, Walter de Gruyter, Berlin, New York, 1995.

[Bo] N. Bourbaki, *Elements de Mathematique, Integration*, 2^{nd} ed., Herman, Paris, 1965.

[E] P. Eymard, *L'algèbre de Fourier d'un groupe localement compact*, Bull. Soc. Math. France **92** (1964), 181-236.

[GM] F. Ghahramani - A. R. Medghalchi, *Compact multipliers on weighted hypergroup algebras I, II,* Math. Proc. Camb. Phil. Soc. **98** (1985), 493-500, **100** (1986), 145-149.

[Je] R. I. Jewett, *Spaces with an abstract convolution of measures*, Advances in Math. **18** (1975), 1-110.

[R] W. Rudin, *Fourier analysis on groups*, Wiley Classics Library, John Wiley & Sons, Inc., New York, 1990.

Department of Mathematics, Tarbiat Modares University,
P.O. Box 14115-175, Tehran, Iran
E-mail: mamini@modares.ac.ir

Pervenuta in Redazione
il 7 aprile 2005 e in forma rivista il 10 ottobre 2007