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## Mathematicians in Bologna in the First Decades After Annexation to the Kingdom of Sardinia

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## Mathematicians in Bologna in the First Decades After Annexation to the Kingdom of Sardinia (\*)

SALVATORE COEN

**Abstract.** – *With the creation of the Italian Kingdom, a new era in mathematical research and teaching in Bologna began, and certainly marked a positive advance with respect to the situation in the preceding decades. As is natural, however, the progress was slow, and passed through many very different stages. We will focus here on this progress in the first decades after 1860.*

The plebiscite of 11-12 March 1860, under the leadership of Luigi Carlo Farini, voted with an overwhelming majority in favor of annexing the *Regie Provincie dell'Emilia (Royal Provinces of Emilia)* of which Farini was the “governor”, consisting of the former *Legazioni pontificie (Pontifical Legations)* (Bologna, Ferrara, Ravenna and Forlì) and the former duchies of Modena and Reggio, and of Parma and Piacenza, to the Kingdom of Sardinia. On 8 March, Farini himself, with a decree law, had established the chairs of Geodetics, Higher Geometry and Applied Mechanics for the *Facoltà Matematica (Mathematics Faculty)* of the University of Bologna. Obviously, this decision had been made in perfect agreement with the central authorities of Turin; the fact, however, of issuing this decree only a few days prior to annexation gives the impression that, even at the local level, it was desirable to emphasize the fundamental importance of the University, and especially its scientific studies, in relation to political power.

On the day of March 17, 1861 when Victor Emmanuel II, King of Sardinia, was proclaimed King of Italy, the Kingdom of Italy was founded. Bologna, of course, belonged to the new state. With the creation of the Italian state, mathematical research and instruction entered a new period of progress, with respect to the situation in the decades immediately preceding the event. As is natural, however, this progress was slow and had to go through numerous, widely differing stages.

(\*) General conference held on September 21, 2011 in the Aula Magna of the University of Bologna on the occasion of the opening session of the XIX Congress of the Unione Matematica Italiana.

In this paper we will illustrate, above all, a possible timeline relative to study and mathematical research as it took place in Bologna in the first century after Italian unity, motivating it with the profound changes that mathematical activity underwent in Bologna during the century. We will then focus more sharply on the initial period, i.e. the twenty years between 1860 and 1880 which, up to this time, have not received much scholarly attention. The topic is very vast indeed; in this paper we will attempt to present as concise an outline as possible.

Three periods can be distinguished rather precisely, starting with the establishment of the Italian Kingdom and ending in the Sixties of the 20th century. The first period, roughly from 1860 to 1880, was one in which the University strove with some difficulty to achieve an authentic resurgence of mathematical studies in Bologna, with ups and downs that we shall describe hereafter in the search for a stability of university instruction and mathematical research it never fully attained. We need only recall that until 1881 the University of Bologna was unable to award the *Laurea (the degree)* in mathematics and could go no farther than a three-year diploma (*licenza*), mainly due to the lack of professors.

The second period was certainly the golden age, and started, in our opinion, around 1880/ 1881 to end abruptly in 1938. That year, Mario Burgatti, Luigi Fantappiè, Beppo Levi and Beniamino Segre were tenured professors in the Science Department in Bologna. Burgatti died in May; the department provided soon after to replace him by calling in Dario Graffi, operational from the following November. In September, with the *leggi razziali (racial laws)*, applied with great zeal, Beppo Levi and Beniamino Segre were expelled. Luigi Fantappiè was already abroad and there he stayed. The *Facoltà di Ingegneria (Faculty of Engineering)*, in turn and also because of the anti-Semitic laws, would lose scientific personalities at the level of Giulio Supino and Emanuele Foà. For several months the *Facoltà di Scienze (Faculty of Sciences)* could not count on the presence of a single professor of mathematics.

This fracture would mark the start of a third stage on the scientific plane, clearly different from those that had preceded it; we can consider this period, roughly, to be between 1939 and 1960. In the Sixties of the 20th century, with the reform of studies for the degree in mathematics and with the advent of mass access to the university, the situation changed again, throughout the country and also in Bologna. This was the period in which construction got underway on the building intended to house the Salvatore Pincherle Mathematics Institute and the Luigi Cremona Institute of Geometry, symbolically destined, in this way, to indicate a new role of mathematical activity at the university.

As we already said, in this paper we will examine the first two stages only. The second is certainly the more exciting one because it marked a solid, continuous rise, but it is the first period, not nearly as well known, which will occupy us here more extensively.

*First period.*

First of all, we have to sketch an outline of the situation in Bologna during the decades preceding annexation, with a few, brief episodes that will illustrate its significance on teaching and on the teachers. Liberal, patriotic ideals had found fertile terrain among the students of the University and also among some professors, as evidenced by the participation of both students and faculty in the uprisings of 1830-31 and later in 1848-49. After the uprising of '31 it would be the famous Austrian foreign minister Klemens Wenzel von Metternich (1773–1859) to recommend the temporary closure of the university; later it was decided (cf. [Simeoni]) that “*the students should continue their studies in the individual cities of residence under teachers chosen by the bishops and approved by the Sacred Congregation. In Bologna, although the tenured professors remained, the courses of the department were to be held in separate locations*”<sup>(1)</sup>. This latter decision had the effect of undermining the very concept of university, of communion, of “*universitas*” between students and professors. The order was revoked and reapplied several times also after the riots of 1848-49. With regard to the students we can recall that during the riots of '48 and '49, the students of Bologna formed a mobile unit that fought alongside the university battalion. Bologna's participation in the Roman Republic was significant; that of Quirico Filopanti was particularly noteworthy, also from the historical standpoint.

A significant episode concerned the professors. In 1835 Ottaviano Fabrizio Mossotti (1791-1863) crossed the ocean, leaving his position of professor in Buenos Aires to take that of professor of Astronomy at the University of Bologna. For political reasons, when he arrived in Italy the position was denied him. We feel that this was a loss that Bologna would have to pay very dearly for. He then obtained a position at the Académie Ionienne in Corfù and, finally, in 1841, moved to the University of Pisa, where he laid the foundations for the extraordinary development of mathematics and physics that would make the University of Pisa a leading player in the decades that followed.

There were other scientists of authoritative stature in Bologna, however, like Francesco Orioli (1783-1856) who taught Physics at the University of Bologna from 1815 to 1831. In 1831 Orioli would be Minister of Education in the “government of the united provinces”. Orioli would also hold an academic position in Corfù. His was actually a very eclectic personality. He was competent in a number of highly different fields such as medicine, chemistry, archeology (particularly Etruscology), politics, ... and is considered to be one of the great tea-

<sup>(1)</sup> *Gli studi dovevano essere fatti nelle singole città sotto maestri scelti dai vescovi ed approvati dalla S. Congregazione. A Bologna, pur restando i professori ordinari, i corsi delle facoltà dovevano aver luogo in sedi separate.*

chers of Silvestro Gherardi (1802- 1879) physicist and historian of science. From 1831 to 1850, he was professor of physics at the University of Bologna. In 1849 he was temporary minister of the Roman Republic. From 1857 to 1861 he was professor of physics at the University of Torino, then director of the Technical Institute of Bologna and later of Florence (1867-1879). The study of hydraulics had a large following in Bologna through the centuries. We can indicate Giuseppe Venturoli (1768-1846) in this connection, well known for his treatise on “*Elementi di Meccanica ed Idraulica*” (Elements of Mechanics and Hydraulics).

Despite the presence of scientific personalities of a certain interest, and studies in hydraulics and astronomy, however, research and also teaching of mathematics in the last decades prior to the annexation of Bologna to the Kingdom of Sardinia was certainly very limited. A profound change of mentality would be necessary in research and instruction.

The chairs established by Farini were assigned, respectively, to Luigi Cremona Geometria Superiore (Higher Geometry), Matteo Fiorini Geodesia (Geodesy) while it was Quirico Filopanti (whose birth name was Giuseppe Barilli) who obtained the chair of Meccanica Applicata (Applied Mechanics).

These appointments were part of a group decided by Terenzio Mamiani della Rovere (1799–1885), who was the Minister of Education of the Kingdom of Sardinia at the time. They were prompted not only by scientific criteria, but also by political criteria: all the new professors professed patriotic sentiments and some, like Cremona and Quirico Filopanti, had participated personally, even fighting among the armed forces in the wars for Italy; many of them would meet again in the Masonic lodges of Bologna. The majority were young men, seldom over thirty. It is important to note that they were scholars who had shown excellent promise but as yet had not produced much, though later, in Bologna, they would take their place as scholars of the first order in the national or international field. Among them, for example, were Giosuè Carducci (1835-1907) appointed to the chair of Italian Eloquence (in 1906 he won the Nobel Prize for Literature), Bertrando Spaventa (1817- 1883) to that of History of Philosophies, Giovanni Capellini (1833-1922) to Geology, Francesco Magni (1828-1887) to Theoretical and Practical Optics and Directorship of the Ophthalmology Clinic of the University of Bologna, Giovanni Battista Gandino (1827-1905) to Latin Literature. With the arrival of the “Piemontesi” (“Piedmonters”) as these new professors were then called, the University of Bologna was destined to undergo some profound changes.

The inaugural lecture by Luigi Cremona, given in November 1860 aroused considerable interest at the time, not only in Italy. Cremona briefly described the difficult condition of mathematical studies in Italy, the “backwardness of the educational programs” (retrivi ordinamenti scolastici), the efforts of a few to raise Italian studies to the level of the more advanced countries in Europe. It was an impassioned, profoundly inspired oration, in the almost incredible period that Italy was experiencing at that time. He concluded with the statement that if the

“*doppia tirannide dello sgherro austriaco e del livido gesuita*” (*dual tyranny of the Austrian brigand and the livid Jesuit*) could have been an excuse of laxness or laziness in the past, the freedom attained obliged us to work hard and maintain vigilance. The newly unified country of Italy had one capital only: Rome “with one king only, Vittorio Emanuele, and one and only one great hero, Garibaldi”. The central point for us here, however, was his outline of the studies of geometry and projective geometry that students would undertake with his guidance, in which he stressed that “*these marvelous, purely geometrical methods have never been taught in our universities*”. The program for the course of Higher Geometry in 1860-61 was as follows: “*Projective properties of geometrical forms. Duality and homography. Applications to the theory of lines and of surfaces of the second order*”, (for which he recommended referring also to the “*Traité de Géométrie Supérieure*” by Chasles). Frankly, this is a rather elementary program that could dampen the enthusiasm deriving from reading his stirring inaugural lecture. Actually – and this is one of the outstanding early merits of Cremona – we have to take account of the fact that in Bologna – and not only in Bologna – before his arrival, geometry was only taught at a rather elementary level in the courses of introduction to calculus. Knowing this, we can better understand Beppo Levi’s remark in a little-remembered commemoration speech ([Levi]) given in Bologna in 1930, “*It is to Cremona himself that we owe the fact that projective geometry has been able to descend from the level of “higher geometry” to the apparently more humble level of a preparatory subject.*”<sup>(2)</sup>. This alone would be no small merit for Cremona. As regards research, at the time of his appointment, Cremona had already published a dozen works especially dedicated to projective geometry in the five preceding years, largely on space curves. One of the first and most important works during his tenure at Bologna is his “*Introduzione...*”(Introduction...) [Cremona, 1862] all of 130 pages long, systematically expounding results and notions on these curves found in Italy and abroad in those years. It would be translated into German by Curtze with other works by Cremona, and published in 1865. Among his more valued writings during his tenure at Bologna we have to remember “*Sulle trasformazioni geometriche...*” (*On Geometrical Transformations ...*) [Cremona, 1863-65] in two notes. His extensive and systematic study in these works on birational transformations and their invariant properties is the reason why these transformations have been known since then, and not only in Italy, as *Cremonian transformations*. A natural outgrowth of the two above-mentioned publications was the later publication “*Preliminari ...*” (*Preliminaries...*) [Cremona, 1866],

<sup>(2)</sup> È al Cremona stesso che dobbiamo se la geometria proiettiva ha potuto discendere dal grado di “geometria superiore” a quello apparentemente più umile di materia propedeutica.

where he set himself the difficult task of generalizing the previous results on three-dimensional space. he writes in the preface: “*The first concept was that of demonstrating synthetically the most essential propositions of higher geometry that pertain to the theory of surfaces of any order, and are expounded analytically or only mentioned in passing in the works of Salmon, Cayley, Steiner, Clebsch,...*” This work and the following one would also be translated by Curtze.

The publication which brings Cremona international recognition, however, is his *Mémoire de Géométrie ...* “ [Cremona, 1868]. Participating in the international competition of the Royal Academy of Science of Berlin, announced in 1864, Cremona demonstrated the results postulated in Jacob Steiner’s publication *Über die flächen dritten Grades* in 1856 and then went farther with his own original results. This publication won him the first prize, ex-aequo with Sturm. It is interesting to note that this success immediately earned him (September 1866) an appointment as an officer of the Ordine dei Santi Maurizio e Lazzaro (Order of Saints Maurice and Lazarus, a very old order of chivalry) to see “*an Italian celebrated also in the battles and victories of science*”<sup>(3)</sup>. It is generally acknowledged that the best period in Cremona’s research was the time spent in Bologna, although it lasted only a few years. By 1866, Cremona had already left Bologna for the Istituto Tecnico Superiore di Milano (now Politecnico di Milano).

In the meantime, in October 1862, Eugenio Beltrami was appointed extraordinary professor<sup>(4)</sup> of Complementary Algebra.

At the time of his appointment Beltrami had published a fifty-page article and two lesser works, but had not earned a degree of any kind. This was enough, however, for Francesco Brioschi, who knew him well, to sponsor his appointment to Bologna. In 1863 Beltrami had already left Bologna for Pisa; formally, this was not a transfer but a promotion to full professor of Geodesy at the University of Pisa. In place of Beltrami, the then extraordinary professor at Cagliari, Pietro Boschi, was appointed extraordinary professor of Complementary Algebra as well as Analytical Geometry<sup>(5)</sup>. Boschi (born in 1833) would die in Bologna in 1887; during his uninterrupted stay in Bologna he did not produce a great deal, but held many courses in different areas of geometry, to the great satisfaction of the students and faculty; he had been a student of Brioschi, and had already been warmly recommended for Bologna by both Brioschi and Cremona as early as 1861. Beltrami would remain in Pisa for three years. There his friend Enrico Betti taught Higher Analysis and Geometry; we should also recall the three long

<sup>(3)</sup> *Il nome italiano celebrato anche nelle battaglie e nelle vittorie della scienza.*

<sup>(4)</sup> The position of “professore straordinario» was not stable and was generally given before that of “professore ordinario»; the latter position gave stability and a salary considerably higher.

<sup>(5)</sup> *Straordinario di Algebra Complementare con l’incarico eziandio della Geometria Analitica.*



visits that Riemann made to Italy between 1862-66, with extended stays in Pisa. Beltrami would return to Bologna as professor of Rational Mechanics in October 1866, a few months after Riemann's death.

Beltrami would remain in Bologna until 1873 when he decided to accept the chair of Mechanics in Rome. The works written by Beltrami while at Bologna are among the most important of his scientific career. In Bologna he wrote his famous "*Saggio di interpretazione della geometria non-euclidea*" (*Essay on the interpretation of non-Euclidean Geometry*) [Beltrami, 1869-1] and "*La teoria fondamentale.*" (*Fundamental theory.*) [Beltrami, 1869-2].

Getting back to the Mathematics Faculty, for the academic year 1862-63 Bologna's faculty could boast Luigi Cremona, Eugenio Beltrami, Domenico Chelini (appointed much earlier, in 1851). The three professors were good friends and held one another in great personal as well as scientific esteem. To this group, we can also add Matteo Fiorini (1827-1901), the illustrious cartographer and historian of cartography, Lorenzo Respighi (1824-1889), the famous astronomer, greatly esteemed also by Schiaparelli and Quirico Filopanti (1812-1894), a highly original personality who, from his chair of Applied Mechanics, taught applied hydraulics, a course conceived for students of engineering. One could think of a sort of miracle: in less than three years, from a situation of grave weakness, mathematics at Bologna became the leading school on the peninsula and this was largely thanks to a political action that was a complete success.

But it was not to be. The period between the second and third wars of Italian independence was a difficult time and there were clear signs of this at Bologna, for example, in Cremona's inability to accept conditions there. His dissatisfaction is obvious in his letters, where he complains bitterly and expresses harshly negative opinions about the situation. I quote here briefly from a letter written in February 1861 to Enrico Betti "*The degradation (with respect to mathematical studies) into which this university has fallen, for many reasons, is such that I see my work as almost futile, in spite of the excellent quality of the students. Moreover, the miserable conditions and gothic regulations of the library do not make it possible for me to study as I used to do in Pavia and Milan.*"<sup>(6)</sup>

Years later, in 1868, Beltrami echoes him (cf. [Brigaglia, Di Sieno, 2010], from the archive of the Mazzini Institute of Genoa) writing to Cremona "*I see an inept rector and a large number of ignorant or unhappy professors (not to sugar-coat it). The few valid ones include several who stink a bit of republican. This is bad, I*

<sup>(6)</sup> *L'abiezione in cui è caduta per tante cause, questa università (rispetto agli studi matematici) è tale che io vedo l'opera mia quasi inutile, malgrado l'ottimo buon valore dei giovani. Di più le misere condizioni e i gotici regolamenti della biblioteca non mi consentono quegli studi, di cui m'ero abituato a Pavia ed a Milano.* The translation is taken from [Brigaglia, Di Sieno].

*admit; but let's be honest, between a republican professor and an ass, which do you consider to be the more dangerous teacher?"* <sup>(7)</sup>

The enthusiasm for the reunification of almost the entire peninsula was accompanied for some by the disappointment of seeing that Italy was not developing the way they would have wanted, in addition to the disappointment of those who had not wanted a reunified Italy in the first place.

What was going on? Full professors were required to swear allegiance to the king and his dynasty. Domenico Chelini and Lorenzo Respighi were not prepared to take such an oath, on account of their allegiance to the Pope. Nor was Filopanti, for his republic ideas. It seemed, in 1860, that a practical solution had been found, as follows. Mamiani interpreted the oath as an obligation only of full professors; he was therefore willing to "demote" (if that is the right word) the full professors to the position of extraordinary professors with full salary but without the oath, and settled the matter that way. In 1864, when the Education Minister was Giuseppe Natoli, from Messina, under Prime Minister Lamarmora, his interpretation of the obligation to swear allegiance became more restrictive and consequently Chelini, Respighi and Filopanti, were fired for their coherence. It should be noted that only four of the full professors refused to swear allegiance at Bologna and that the fourth, the zoologist and mineralogist Bianconi, was also on the Mathematics Faculty. At the insistence of his students, Filopanti, was offered the position of "lecturer" in 1866, but in 1868 his political beliefs prompted him to leave this position as well. In a much-belated display of remorse, both Filopanti and Respighi were appointed "honorary professors" in 1878; the other two expelled professors died at about the same time in 1878. We should also note that Filopanti had already been appointed professor of Mechanics and Hydraulics in 1848, and dismissed following his particularly active participation in the Roman Republic in 1849.

After the appointment of Beltrami in 1862 and until 1873, there were no other appointments aside from that of Boschi. Thus, after 1864, the teaching staff was reduced and both instruction and research suffered greatly. Beltrami's return in 1866 was offset by the absence of Cremona, who would go to teach at the Higher Technical Institute of Milan. He formally maintained his position in Bologna, however, so that the university could not dispose of his chair.

In October 1873 Beltrami moved to Rome; in his place, Cesare Razzaboni was brought in from Rome. In 1874 Razzaboni was called back to Rome to direct the Institute of Hydraulics at the local School of Application for Engineers.

<sup>(7)</sup> *Vedo un rettore inetto, ed un gran numero di professori ignoranti o tristi (non dorianamo le pillole). Nel piccolo numero dei valenti ce ne sono alcuni che puzzano un po' di repubblica. È un male, lo confesso; ma, siamo sinceri, fra un professore repubblicano e un professore asino quale reputi più pernicioso all'insegnamento?"*

At this point a curious and forgotten episode may be of some interest, also because of its unusual aspects.

On 29 August 1874, the city council of Bologna, writes to the Rector that it had been “*informed by the press of the transfer of Razzaboni to Rome; if this should be confirmed, you should at least bring Beltrami back*”<sup>(8)</sup>.

The Rector replies on 2 September thanking the council for having expressed “*the disapproval of the entire city in seeing the University reduced to little more than a railroad station where the Professors are sent for their trial period, then called to another university at the first opportunity, not for reasons of study, but to follow particular ideals and undermine the foundations of this ancient school.*” “*As regards Beltrami it is unthinkable*”<sup>(9)</sup>. He expresses the hope that others can be called and can “*help lessen the gravity of the loss of the excellent Razzaboni*”<sup>(10)</sup>.

The council, encouraged by the letter, then writes to the Prime Minister Marco Minghetti (1818-1886), who was from Bologna, expressing the same preoccupation.

The interim minister Gerolamo Cantelli replies on 21 September 1874 that the “*call to the School of Application in Rome (of Razzaboni) is nothing more than the confirmation of a project that has long been in the offing and was almost impossible to deny*”<sup>(11)</sup>. At the same time, he would do whatever he could to appoint another full professor of Rational Mechanics.

And so, in December of that same year Ferdinando Paolo Ruffini (1823-1908) was transferred from the Chair of Differential and Integral Calculus of Modena to that of Rational Mechanics in Bologna. He would produce about fifty publications on various subjects, with a prevalence of works of geometry, mechanics and history. He was the first director of the *Scuola di Magistero (School of Education)*, and would be chairman of the Faculty of Sciences and the only mathematician to be Rector of the University of Bologna after annexation.

After noting that the “*practical course for engineers,*” while it can be shown to be useful for the training of agronomists, is certainly not able to prepare good

<sup>(8)</sup> *di essere stata informata dalla stampa del trasferimento di Razzaboni a Roma; se questo dovesse essere confermato, almeno si provveda a far tornare Beltrami.*

<sup>(9)</sup> *disapprovazione dell'intera città nel vedere l'Università nostra ridotta quasi ad essere una stazione ferroviaria ove i Professori si mandano a fare le loro prove, per richiamarli in altra università alla prima occasione non già per l'opportunità degli studi, ma per seguire particolari propositi, e minare le fondamenta di questo antico studio.... In quanto al Beltrami non vi è da pensarvi.*

<sup>(10)</sup> *far sentire meno grave la perdita dell'egregio Razzaboni.*

<sup>(11)</sup> *richiamo presso la Scuola d' Applicazione di Roma (di Razzaboni) non è quindi che il conferimento di un disegno già da tempo formato ed era quasi impossibile farne a meno.*

engineers, we come, after various and exhausting events, to 1877, with the establishment of the *Scuola di Applicazione per Ingegneri (School of Application for Engineers)*, supported by a local consortium of which the municipality, province and other local agencies are members. This was probably the most appropriate answer to the hemorrhage of scholars migrating to the School in Rome, and immediately encountered the hostility of everyone in Rome. It is perhaps indicative that it was not until 1897 that the School was allowed to add the title of “Royal” to its name. Cesare Razzaboni came back to Bologna from January 1, 1877, to occupy the chair of Infinitesimal Calculus and was appointed to provide for establishing the new school. In November of that year, 1877, the School was officially established and Razzaboni, having transferred to the chair of Hydraulics, was its director.

Cesare Razzaboni (1827-1893) had graduated with a degree in theoretical engineering as well as mathematics and physics from the University of Modena after attending the Scuola dei Cadetti Matematici Pionieri (a Modenese university institute to prepare students in engineering and architecture). His scientific production concentrated on problems of theoretical hydraulics and, especially in his later life, also experimental hydraulics.

The establishment of the School of Application of Bologna led to the creation of new chairs.

In August 1877 Luigi Donati (1846-1932), former full professor of Physics at the Higher Technical Institute of Milan was appointed extraordinary professor of Technical Physics at the School of Application of Bologna. In 1879 he would also become extraordinary professor of Mathematical Physics at the of Sciences. Donati, self-taught prior to his university studies, won the Lavagna prize and graduated from the Scuola di Magistero at the Normal School of Pisa, where he was then assistant to Riccardo Felici (1819-1902). He produced about fifty publications on elasticity (with important contributions to the Menabrea Theorem), on Electromagnetism, and on vectorial fields, which he was one of the first to teach in Italy. He was an excellent teacher and wrote, among others, “*Appunti didattici sulla teoria della relatività (relatività ristretta) (Notes on teaching the theory of relativity) (special relativity)*” in 1922.

Another call, in August 1877, was to Pietro Riccardi (1828-1898) a professor of Practical Geometry at the School of Application. Riccardi, who was originally from Modena, had taught Theoretical and Practical Geodesics and later Analytical Geometry at Modena. He had graduated from that University after attending the Scuola dei Matematici Pionieri. He published a great deal on various subjects and also practiced the engineering profession for many years in Modena. He is mainly remembered today for his activity as a historian of mathematics. Perhaps his most famous work is the “*Biblioteca matematica italiana dalle origini della stampa sino ai primi anni del secolo XIX (Italian mathematical library from the origins of printing to the 19th century)*” in two

volumes, completed later by various appendices, corrections and additions, as well as his "*Cenni sulla storia della geodesia in Italia dalle prime epoche fin oltre alla metà del secolo XIX*" (Brief overview of the History of Geodesy in Italy from the early epochs until the mid-19th century), which he wrote while he was at the University of Bologna.

We cannot go more deeply into the details of the events at the School of Application though they could be of interest to us, not only for the positions of the traditional mathematics course, but also for those of Geodesy and Hydraulics.

Our focus here, however, is the Faculty of Sciences which continued to be a sort of railroad station. In 1875 the young Sardinian Francesco Flores d'Arcais (1830-1890) was called from the chair of Infinitesimal Calculus in Cagliari to replace Razzaboni, but the following year he would return to Cagliari in a sort of exchange with Antonio Fais (1841-1925) also from Cagliari. Fais was appointed extraordinary professor of Algebra and Analytical Geometry at Bologna and assigned the chair of Graphic Statics. He would then pass to Infinitesimal Calculus, but three years after his appointment he went back to Cagliari. While at Bologna he wrote interesting works on classical analysis and arguments of differential geometry.

The report of Rector Luigi Calori for the academic year 1876-77 was devoted, to a large extent, to the difficult situation of Mathematics at Bologna, where it was evidently the weak point of the whole university.

Riccardo de Paolis was called to the chair of Algebra and Geometry in 1878 (he had been assistant, before then, in Rome), but in 1880 he would be appointed by competition to the chair of Higher Geometry in Pavia.

So in 1880 Bologna was in a difficult position again after the simultaneous transfers of Fais and De Paolis.

We can try to summarize this period 1860-80 with a few considerations. It starts with a period of influence (one might jokingly call it a "colonization") on the part of Pavia which brought the mathematicians Cremona, Beltrami and Boschi, all trained at Pavia, to Bologna. Later there was a period of intense exchanges with Cagliari and finally a sort of "Modenese colonization" (Ruffini, Razzaboni, Riccardi) of the School of Application in Bologna. The contribution of locally trained mathematicians was almost nil.

Evidently, for pure mathematicians Bologna was not a choice posting, although it was at Bologna that Cremona wrote his best works, and Beltrami also has a very active period there scientifically, much more so than at Pisa. We have to add, however, as further proof of what we have said that when the chair of Higher Analysis became free in 1877, and in 1878 that of Mathematical Physics (the only applicant, Bruno Padelletti having withdrawn after being appointed Visiting Professor of Rational Mechanics in Palermo), not a single candidate applied.

For several years, Bologna had been considered with some regard, but that all came to an end. The feeling almost of offense experienced by the city after

Razzaboni's transfer was justified. In the long run, the finest mathematicians who passed through Bologna all ended up, around 1874, at the School of Application in Rome, without forgetting that Chelini and Respighi were also in Rome, though in different situations.

One might ask how it was that so many scholars highly sensitive to their civic and institutional duties could show so little interest in the local institutions. Naturally, then as now, personal ambition played a part. There may be another reason, however. In the early years we examined, these appointments were generally made by order of the ministry, without competition and without calls. In this way, the professors felt a duty to respond more to the national interest, represented by the ministry, rather than the local interests. If this interpretation should be correct, the ministers would have to take the blame for the many transfers from Bologna. Effectively, a careful reading of the records proves it, at least in some cases. When Cremona was "ordered" to Milan, the minister Domenico Berti wrote that Higher Geometry at Bologna was no longer a required course, while "*In the Royal Higher Technical Institute of Milan, destined to prepare the finest engineers and strongest mathematicians, the needs of the students reflect the contribution of that teaching provided by a top professor like Cremona*"<sup>(12)</sup>.

When Beltrami was transferred to Roma, the minister at the time, Antonio Scialoja, wrote on 20 October 1873 "*The Ministry hesitated for some time, greatly regretting the need to remove such a fine man from your University. But then it felt it could provide a valid replacement for the chair left vacant by Beltrami. Effectively, Professor Cesare Razzaboni...*"<sup>(13)</sup> Perhaps that is also why when Razzaboni was transferred only a year later, Bologna felt cheated.

What is even more important, we have to add, is that in all those years Bologna did not manage to award a degree in mathematics to a single student, because it was unable to hold the fourth year courses.

Quite frequently in Cremona's letters, alongside his criticisms of certain colleagues, we read praises for his young students. At least two names of students in the period we are examining stand out. I refer to Eugenio Bertini and Gregorio Ricci Curbastro. Bertini had the good luck to be able to follow the last lesson of Higher Geometry held by Cremona who wrote of him in May

<sup>(12)</sup> *Nel R. Istituto Tecnico Superiore di Milano destinato a preparare valenti ingegneri e robusti matematici, le esigenze degli studenti riecchieggono l'aggiunta di quell'insegnamento impartito da un uomo valente quale è il Cremona.*

<sup>(13)</sup> *Il Ministero rimase a dir vero alcun tempo dubbioso, assai spiacciato di togliere a codesta Università un così chiaro uomo. Se non che ha poi creduto di poter provvedere tale partito, essendosi presentata l'opportunità di provvedere contemporaneamente e nel modo più convenevole alla cattedra che avrebbe costì lasciata vacante il Beltrami. Di fatti il prof. cav. Cesare Razzaboni...*

1866 in a letter to Chelini “*he kept up with everything*”<sup>(14)</sup> ([Enea, Gatto]); that same June, Bertini enrolled as a volunteer and fought in a Garibaldi corps near Trento; in November he passed the examination of Higher Geometry with Cremona, Beltrami and Domenico Piani on the examining committee, with a grade of 30 out of 30 and distinguished honors. He would then go to Pisa to finish the last year. Gregorio Ricci was admitted to the second year, in view of the studies he had already completed at the university in Rome. He studied at Bologna for two years and after passing the third year was admitted to the Normal School of Pisa. I should note here that many young mathematics students at Bologna interrupted their studies in Bologna to continue at the Normal School of Pisa. Mario Pieri, for one, attended at Bologna for just one year in the academic year 1880-81, then entered the second year at Pisa. Later, another example is that of Giuseppe Vitali, who switched to the Normal School in 1897 after two years in Bologna; it seems that this custom was actually encouraged by the professors at Bologna.

### *Second period.*

Let us return now to the general situation at the Faculty of Sciences. We have seen that in 1880 Fais and De Paolis left Bologna at the same time. While Mathematical Physics was able to continue with the possible aid of the professors of the School of Application, geometry risked having only a single chair, Pietro Boschi, burdened with excessive teaching obligations. Analysis was in even worse shape. We should recall that Weierstrass had been at Berlin since 1857 and that these were the most intense years of his great school, but it is extremely difficult to find any trace of his methods at Bologna.

This time, however, there is a positive turn of events when that very same year, 1880, Cesare Arzelà (1847-1912) was called to Bologna followed, a few months later, by Salvatore Pincherle (1853-1936). Arzelà had graduated from Pisa in 1869 as a student of the Normal School. For some time he had taught in secondary schools, won a scholarship for advanced training abroad which he decided to forgo, and been appointed to Palermo to the chair of Algebra. At Bologna he would teach Infinitesimal Calculus and would remain there until his death in 1912. This excellent teacher had among his students Leonida Tonelli, Filippo Sibirani (1880-1957) and others. One of his most important works was the paper *Sulla serie di funzioni* ([Arzelà] “On series of functions”) published in 1899. He is well known for his works aiming to prove the Dirichlet Principle by a direct method.

<sup>(14)</sup> *ha tenuto dietro a tutto.*

Finally, from the academic year 1881- 82 Bologna was able to complete the four-year curriculum of studies for a degree in Mathematics. For years, Arzelà would hold the course in Higher Analysis and Pincherle that of Higher Geometry.

In 1888 Bologna was the site of a Universal Exposition and celebrations of the eighth centennial of the University. The mathematicians seem to have been kept rather out of sight during the celebration, but on that occasion the Faculty of Sciences, evidently at the prompting of the mathematicians, awarded several degrees “honoris causa” to such luminaries as Arthur Cayley, Charles Hermite, Felix Klein, Leopold Kronecker, Gosta Mittag-Leffler, Nathaniel Pringsheim, Lord Kelvin and Karl Weierstrass.

That same year, 1888, Domenico Montesano (1863-1930), one of Cremona’s students, was called by competition to the chair of Projective and Descriptive Geometry (*Geometria Proiettiva e descrittiva con disegno*), to which he came from Rome; Montesano would be transferred early in December 1893 after winning a competition for the chair of Geometry in Naples and did not have time to leave tangible signs. In 1894 a very young Federigo Enriques (1871-1946) was appointed to the chair; he was appointed extraordinary professor from 1896. The appointment was not without some problems considering that another illustrious geometrician, Mario Pieri (1860-1913), who had also studied briefly at Bologna was interested in the same position (cf. [Marchisotto, Smith], ch. 1.15, *The Bologna affair*). Enriques who produced what were perhaps his most interesting works on geometry, had other interests ranging far beyond geometry. For example, in April 1911 Enriques was the organizer and Chairman of the International Congress of Philosophy, held in Bologna. Enriques marked a profound change in research and teaching in Bologna. We cannot go more into detail on this important personality, however considerable information about him can be found in the volume mentioned previously [Coen]<sup>(15)</sup>.

For mathematical physics we have to remember the call to Pietro Burgatti (1868-1938), from Messina, in 1908; he had worked in Rome under the guidance of Eugenio Beltrami and Valentino Cerruti. Burgatti was an acclaimed mathe-

<sup>(15)</sup> In the volume [Coen] the scientific personalities of the following mathematicians who worked in Bologna are especially illustrated: Ugo Amaldi, Eugenio Beltrami, Enrico Bompiani, Roberto Bonola, Pietro Burgatti, Lamberto Cattabriga, Gianfranco Cimmino, Luigi Cremona, Federigo Enriques, Dario Graffi, Beppo Levi, Salvatore Pincherle, Bruno Pini, Beniamino Segre, Leonida Tonelli, Tullio Viola, Giuseppe Vitali, Giulio Vivanti (alphabetical order). The contributors are Nicola Arcozzi, Maria Teresa Borgato, Umberto Bottazzini, Aldo Brigaglia, Ciro Ciliberto, Salvatore Coen, Simonetta Di Sieno, Mauro Fabrizio, Paolo Freguglia, Paola Gario, Livia Giacardi, Sandro Graffi, Angelo Guerraggio, Michel Guillemot, Ermanno Lanconelli, Gabriele Lolli, Erika Luciano, Francesco Mainardi, Pietro Nastasi, Gianni Pagnini, Clara Silvia Roero, Enrico Rogora, Irene Sabbadini, Emma Salent del Colombo, Edoardo Sernesi, Daniel C. Struppa, Rossana Tazzioli, Sergio Venturini (alphabetical order).



mathematical physicist who also worked in the field of astronomy. If Arzelà at Bologna represented what we can now call Real Analysis, Pincherle represented Complex Analysis. Like Arzelà he was a graduate of the Normal School, with a degree in Mathematical Physics from Pisa with Enrico Betti, and had taught in the secondary schools of Pavia collaborating, at that time, with Casorati; he had won a position for advanced training in Berlin under Weierstrass in the academic year 1878-79, and was then appointed extraordinary professor at Palermo. He would be called to Bologna in December 1880 as Professor of Algebra and Analytic Geometry. In 1912, on Arzelà's death, he would transfer to Infinitesimal Calculus. Already by 1900 he had already published 105 works. At the end of his career, between articles, books, etc. his scientific publications totaled around 270.

The time he spent in Berlin had a great influence on him; he published the re-elaborated text of the course of introduction to the theory of analytic functions held in Berlin by Weierstrass. He studied the development of analytic functions in series of particular functions and thus encountered delicate new problems of convergence, which he solved by original methods of unquestionable interest and novelty. In 1897-98 he introduced the notion of *adjoint operator* of a given linear operator on some vector spaces of analytic functions. His studies were noticed by A. Hurwitz who, at the second conference of the Zurich congress devoted to the development of the theory of analytic functions, mentions Volterra's line functions and observes how Pincherle, Levi Civita and Bourlet were also moving on this line of thought which studied the "functions of functions"<sup>(16)</sup>.

Pincherle's studies then led him to a systematic study of the theory of distributive operations and their applications to analysis, which he discusses in his book [Amaldi, Pincherle], written with his student Ugo Amaldi. These works reveal that he can be considered a precursor of the theory of complex linear operators. He then studied a number of other problems, such as problems of iteration. From the Twenties, while continuing his scientific activity, he devoted considerable time to resolving problems of an organizational and political nature in local, national and international research. It was Pincherle who founded the UMI and was its first president; he was the founder and first director of the UMI bulletin and the Mathematics Institute of Bologna. None of these efforts was easy. In 1924 at the International Congress of Mathematicians in Toronto, where they were discussing the possibility of readmission of mathematicians from Germany,

<sup>(16)</sup> *Von mathematisch-physikalischen Vorstellungen ausgehend gelangt Herr Volterra dazu, Funktionen von Linien zu untersuchen, d.h. solche Abhängigkeitsgesetze, welcher jeder Linie im Raume einen bestimmten komplexen Zahlenwert zuordnen. Eine ähnliche Ideenbildung liegt neueren Untersuchungen von Pincherle, Levi-Civita und Bourlet zu Grunde. Hier werden solche Gesetze betrachtet, die aus einer beliebig angenommenen Funktion eine neue Funktion entstehen lassen. Man hat es, in einem höheren Sinne des Wortes, mit Funktionen von Funktionen zu thun. ([Hurwitz], p. 106)*

Austria, Bulgaria and Hungary to the IMU (International Mathematical Union) congresses, which he attended as speaker at one of the general conferences, he took a position favorable to readmission and in 1926 was appointed to organize the international congress at Bologna in 1928. Pincherle is the only Italian thus far to have been president of the IMU. Aside from the difficult problems of organization, he also faced serious political problems in view of the reluctance of several German mathematicians to participate in the Congress, where they continued to feel ostracized, and of the obstinacy of others who were effectively against their readmission. Aided by his colleague at Bologna, Umberto Puppini (1884-1946) scholar of hydraulics, Director of the School of Application, Pincherle managed to keep a firm hold on the situation and resolved the problem by having all the participants in the Congress invited by the University of Bologna. Other problems concerned relations between the UMI and the CNR (Consiglio Nazionale delle Ricerche). The scientific committee of the UMI was also the mathematical committee of the CNR. It was decided that this committee should be independent of the UMI and thus it was possible to proceed with the organization of the Congress, without obstacles, but also without any support from the CNR. The congress at which Pincherle had been called to president had as vice presidents, among others, Hadamard and Hilbert, and was a great success from the scientific and organizational standpoint, as proven by the volumes then published; there were 76 German and 51 French mathematicians, with a total of more than 1,100 participants.

In the meantime, in 1923, to be exact, Federigo Enriques, had gone to teach in Rome replaced, at the request of Castelnuovo, by Enrico Bompiani (1889-1975), then at the Politecnico di Milano; Bompiani would stay in Bologna only four years, however, until November 1927, producing about thirty publications in that time. From 1931 he would be replaced by a young (28) Beniamino Segre (1903-1977), called to the chair of Higher Geometry. On the death of Arzelà in 1912, we immediately see new courses, which Leonida Tonelli was asked to teach (1885-1946) as, after the World War I and his call to Parma, he would be in Bologna in 1922 with the chair of Higher Analysis. Tonelli would leave Bologna for Pisa from the academic year 1930-31, replaced in Bologna by Giuseppe Vitali (1875-1932) above all through the intercession of Beppo Levi (1875-1961) who, in the meantime, in 1928, had been called to the chair of "Elements of the Theory of Functions". Vitali died in 1932 and was replaced that same academic year 1932-33 by Luigi Fantappiè (1901-1956) who would only teach briefly in Bologna, however, on account of his prolonged stays abroad.

At the time of the call to Bologna, Beppo Levi was at the peak of his career. He had already achieved important results in Mathematical Analysis with a well-known work on the principle of Dirichlet (1906), in Algebraic Geometry with his results regarding the singularity of algebraic surfaces, in number theory, as well as in Mathematical Logic. The closure of the Faculty of Chemistry at Parma, of

which he was chairman, forced him to move. At Bologna, he taught differential equations, mathematical logic and, for the first time, he studied problems of the theory of one and several complex variables. He also wrote his well-known treatise of Mathematical Analysis while at Bologna. Giuseppe Vitali came to Bologna when he had already been stricken by hemiplegia and had only two years of work ahead of him. In those years, however, he was very active, with studies in differential geometry in Hilbert spaces, and extended his research to questions of the physical character and structure of matter; his treatise on the theory of the functions of real variables was not finished and would later be completed by Giovanni Sansone (1888-1979).

Leonida Tonelli had received his scientific training at Bologna; before being called to teach there he had been professor at the universities of Cagliari and Parma and had experienced considerable hardship during the war. While at Bologna his main research was on the Calculus of Variations. When he left Bologna he had produced about ninety publications, including his important treatise on *Trigonometric Series* (Zanichelli, 1928). Though very young, Beniamino Segre came to Bologna having already published more than forty works and, while at Bologna, he wrote another fifty or so. These were mostly studies of Algebraic Geometry but he also studied Differential Geometry and showed an interest in the Theory of Functions of Several Complex Variables. Among his students he was considered to be very strict. It seems that the application of the racial laws was a hard blow for him, also from the psychological standpoint.

Reflecting on this second period, we could say that there had been another “colonization”, this time successful, by Pisa. Effectively Donati and Flores d’Arcais first, and later Arzelà, Pincherle and Enriques all came from the Normal School. The fact that most of the then young professors - Arzelà, Pincherle and Burgatti - came to Bologna with the idea of staying there, was also a fundamental fact. It would give particular stability to studies there and would prompt the professors to seek out the best colleagues. It was in this period that many excellent researchers got their training, and would go on to raise the quality of other schools. We are thinking of Giulio Vivanti (1859-1949), Carlo Severini (1872-1931), Roberto Bonola (1874-1911), Ugo Amaldi (1875-1927), Oscar Chisini (1889-1967), Antonio Mambriani (1898-1989), Tullio Viola (1904-1985), Silvio Cinquini (1906-1998). Actually, there were very few full professors of mathematics left of those who had trained at Bologna, like Ettore Bortolotti (1866-1947), who contributed so much to the knowledge of the History of Mathematics at Bologna, particularly with his studies of the Renaissance period, or like Leonida Tonelli.

Numerous publications deal with mathematics at the University of Bologna. As a general indication we will limit ourselves to mentioning the well-known book by Ettore Bortolotti [Bortolotti] and the more recent [Coen].

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