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On the hypothesis that the expansion of the Universe is due to a distribution of neutrinos of sufficiently high density and on the gravitational redshift

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SEZIONE II

(Fisica, chimica, geologia, paleontologia e mineralogia)

Relatività. — *On the hypothesis that the expansion of the Universe is due to a distribution of neutrinos of sufficiently high density and on the gravitational redshift.* Nota (*) del Socio GLEB WATAGHIN.

RIASSUNTO. — Partendo dalle equazioni dell'Universo in espansione di Einstein, con la « costante cosmologica » $\Lambda > 0$ corrispondente a « fluttuazioni del vuoto », si propone una nuova interpretazione del « parametro di espansione » di Hubble.

The purpose of this Note is to study a model of expanding, spatially homogeneous and isotropic universe of finite 3–dimensional volume $2\pi^2 R^3 (x_4)$, from a partially new point of view. The integration of Einstein's gravitational equations, with the cosmological constant $\Lambda > 0$, will lead us to an interpretation of the Hubble effect, which, as far as I know, was not considered before.

Starting from the form of the line-element ds^2 suggested by Friedman in his pioneer work [1]:

$$d\tau^2 = \frac{ds^2}{c^2} = -\frac{R^2(x_4)}{c^2}[dx_1^2 + \sin^2 x_1 \, dx_2^2 + \sin^2 x_1 \sin^2 x_2 \, dx_3^2] + g_{44}(x_4)\, dx_4^2$$

$$(1) \qquad 0 \le x_1 \le \pi \quad ; \quad 0 \le x_2 \le \pi \quad ; \quad 0 \le x_3 \le 2\pi$$

we shall introduce following assumptions:

1. There was in the early stage of the evolution a moment when matter and radiation were at very high temperature ($\gtrsim 10^{12}\,{}^0$K) and high density ($\rho \gtrsim 10^{13}$ gr/cm^3). One can describe approximately the evolution at this epoch as a succession of nearly equilibrium states of relativistic particles obeying the Einstein gravitational equations and the approximate equation of state: $p(x_4) = \frac{1}{3}\rho(x_4)c^2$ (in " cosmological coordinates " $x_1 x_2 x_3 x_4$).

The special choice of the coordinates in (1) allows to represent the radius of the 3–dimensional space $R(x_4)$ the density $\rho(x_4)$ the $g_{44}(x_4)$ component of the gravitational potentials, the temperature $T(x_4)$ and some other parameters as functions of the " cosmological time " x_4 only.

2. Besides the assumption of initial temperature T_i and density ρ_i at an arbitrary chosen epoch $x_4^{(i)}$, we want to add the hypothesis that the pair production of antiparticles at temperatures $T \gtrsim 10^{12}$ (^{0}K) played such a

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dominant role that the 4 conserved quantities: the total charges, the baryonic number B, and 2 leptonic numbers L_e, L_n are nearly or exactly vanishing (see [2]). After some time, the cooling and espansion lead to the separation from the rest of the matter of the $\nu_e\, \tilde{\nu}_e\, \nu_\mu\, \tilde{\nu}_\mu$ neutrinos, when the mean free path of these particles becomes comparable with the dimensions $R(x_4)$ of the Universe. Later on, the photon-phase separated, forming what is now known as the 3 (^{0}K) blackbody radiation. Since the initial energy density $(\rho_\nu^{(i)})\, c^2$ of neutrinos at e.g. $T_i \sim \frac{m_\pi c^2}{k} \sim 1{,}5 \cdot 10^{12}$ (^{0}K) is of the same order of magnitude as the energy density of other particles [2], and neutrinos, after separation, are subject to an adiabatic expansion, whereas other particles suffered in later time annihilation processes, formation of nuclei, stars and galaxies and photon-absorption processes, we conclude that the present day density of neutrinos is greater than the average density of matter and we shall assume that at the present epoch $\rho_\nu^{(0)} \gtrsim 10^{-29}$ gr/cm^3. (See [3] and a forthcoming Note of A. Wataghin and A. Agnese). Therefore, after the initial interval of expansion the neutrinos became the main cause of the expansion of the Universe, as was suggested recently [3]. In the following we shall treat the above equation of state and the Einstein gravitational equations in Friedman form:

$$3\,\frac{\dot{R}^2}{R^2}\,\frac{1}{g_{44}} + 3\,\frac{c^2}{R^2} = \Lambda + 8\pi G\rho \tag{2}$$

$$2\,\frac{\ddot{R}}{R}\,\frac{1}{g_{44}} + \frac{\dot{R}^2}{R^2}\,\frac{1}{g_{44}} + \frac{c^2}{R^2} - \frac{\dot{R}}{R}\,\frac{\dot{g}_{44}}{g_{44}^2} = \Lambda - \frac{1}{3}\,8\pi G\rho \tag{3}$$

referring to the neutrino filled cosmos ($\rho = \rho_\nu$). In order to integrate these equations we begin with a familiar procedure (multiplying (2) by $\frac{1}{3} R^3 g_{44}^{1/2}$ and differentiating; multiplying (3) by $g_{44}^{1/2} R^2 \dot{R}$ and substracting) to obtain the relation:

$$\frac{\dot{\rho}}{\rho} + 4\,\frac{\dot{R}}{R} = 0 \qquad \text{or} \qquad \rho = \frac{A}{R^4} \cdot \tag{4}$$

One can now establish the following relation between $R(x_4)$ and $g_{44}(x_4)$: $g_{44}^{1/2}(x_4) = bR(x_4)$, where $b = \text{const}$. Let us consider in each point an infinitesimal domain D_L, where we can suppose $g_{\mu\nu}$ constant and can choose a Lorentz reference system in a way that the origin of spatial coordinates coincides with the centre of momenta of all particles of the domain D_L, and the time axis is parallel to the direction of the cosmological time x_4. The red shift of distant stars is also isotropic in such a frame (Dirac). Considering an atomic clock at rest in the centre of momenta of the domain D_L (with $x_1 = x_2 = x_3 = 0$), one finds easily that, in order to agree with the result concerning gravitational red shift obtained with Mössbauer effect and with astronomical observations, one must assume that the " naturally measured time interval " $d\tau$ is given by (1): $d\tau = g_{44}^{1/2}(x_4)\, dx_4$ and the rate of retardation of this clock is measured by $g_{44}^{-1/2}$ (we suppose $|g_{44}| < 1$ in distant past and $g_{44} \rightarrow 1$ in the distant future). One arrives at a similar conclusion considering the wave length λ of e.g. a stationary wave: if it arises from identical atoms at rest in two different instants of time x_4 and x_4', one has for λ in two points $\lambda : \lambda' = R(x_4') : R(x_4)$. The relation $\frac{\nu}{\nu'} = \frac{\lambda'}{\lambda}$ follows from the accord of the red-shift measurements with the Mössbauer effect and by means of the astronomical observations. It corresponds to a constant value of the speed of neutrinos and of light signals. One deduces from this relation and from (1):

$$g_{44}^{1/2}(x_4) = b\mathrm{R}(x_4) = \mathrm{R}(x_4)/\mathrm{R}_0 \tag{5}$$

where $b = \mathrm{R}_0^{-1} = \text{const.}$

One obtains (5) also analyzing the equations of the geodetics of the light signals or of the neutrinos.

One deduces also from the equation of state of the neutrino pairs at the epoch when the neutrino density greatly dominates all other densities, that the scalar invariant of the matter tensor vanishes: $\mathrm{T} = \mathrm{T}_\mu^\mu = 0$ and the scalar Rieman curvature $\mathbf{R}$ is constant: $\mathbf{R} = 4\Lambda = \text{const.}$ All these and the following deductions have obviously the character of an approximation. We do not e.g. quantize the gravitational field and we do not consider the possible contribution of the gravitons.

One can now eliminate from (2) and (3) either $\mathrm{R}(x_4)$ or $g_{44}(x_4)$ and one obtains, remembering (4) and the relation which follows from (5):

$$\frac{1}{2}\frac{\dot{g}_{44}}{g_{44}} = \frac{\dot{\mathrm{R}}}{\mathrm{R}} :$$

$$\frac{3}{4}\frac{\dot{g}_{44}^2}{g_{44}^3} + 3\frac{c^2 b^2}{g_{44}} = \Lambda + 8\pi \mathrm{G}\,\mathrm{A}\, b^4 g_{44}^{-2} \tag{2'}$$

One has also:

$$\mathbf{R} = 4\Lambda = \frac{6}{\mathrm{R}^3 b^2}(\ddot{\mathrm{R}} + b^2 c^2 \mathrm{R}) = \text{const.} \tag{2''}$$

We adopt the interpretation of the Hubble effect as due to the gravitational red shift. If ν and ν' indicate the frequencies of two identical atom-clocks at two epochs x_4 and x_4', one has:

$$\frac{\nu}{\nu'} = \frac{g_{44}^{1/2}(x_4)}{g_{44}^{1/2}(x_4')} = \frac{\mathrm{R}(x_4)}{\mathrm{R}(x_4')} \tag{6}$$

The neutrino filled Universe has the total-particle energy:

$$\mathrm{E} = 2\pi^2 \mathrm{R}_0^3 \rho_0 c^2$$

at the epoch x_4° when $g_{44}(x_4^\circ) = 1$, and the gravitational energy is assumed to be negligible. At an epoch with $g_{44}(x_1) < 1$, when the particle energy is red shifted: $\varepsilon_\nu = h\nu\, g_{44}^{1/2}(x_4)$, one obtains for the total energy of neutrinos interacting only with the gravitational field the following expression:

$$E = 2\,\pi^2 R^3 g_{44}^{1/2} \rho c^2 = 2\,\pi^2 A \frac{g_{44}^{1/2}}{R} c^2 = 2\,\pi^2 A b c^2 = \text{const.}$$

We conclude that assuming the conservation of the number of the neutrinos during the adiabatic expansion, we can consider that E represents the total energy of neutrino-particles + gravitational energy. The sum is conserved, in an adiabatic expansion, whereas the particle energy $2\,\pi^2 R^3 \rho c^2$ and the temperature T varies as $R^{-1}(x_4)$ or as $g_{44}^{-1/2}(x_4)$.

In order to give an example of a numerical integration of (2) we chose following values of the constants:

$$b = R_0^{-1} = 10^{-28}\ \text{cm}^{-1} \quad , \quad \Lambda \cong 3 \cdot 10^{-36}\ \text{sec}^{-2} ,$$

$$A b^4 = \rho_0 R_0^4 b^4 \sim \rho_0 \sim 2 \cdot 10^{-29}\ \text{gr/cm}^3 .$$

Indicating: $z = g_{44}^{1/2} = bR$ and integrating (2′), one has

$$2{,}9 \cdot 10^{-18} \int_{x_4^i}^{x_4} dx_4 = \int_{z_i}^{z} \frac{dz}{\sqrt{1 - 0{,}98\, z^2 + 0{,}119\, z^4}} \tag{7}$$

or, indicating:

$$z = l\xi \qquad l = 1{,}096 \qquad \varphi = \arcsin \xi$$

$$k = \sin\alpha = 0{,}4144 \qquad \alpha = 24^0\, 29'$$

one obtains

$$2{,}9 \cdot 10^{-18}(x_4 - x_4^{(i)}) = l \int_{\xi_i}^{\xi} \frac{d\xi}{\sqrt{(1-\xi^2)\,(1-k^2\xi^2)}} \cong 1{,}096 \int_{\varphi_i}^{\varphi} \frac{d\varphi}{\sqrt{1 - k^2 \sin^2\varphi}} . \tag{7'}$$

Here the " initial time " is referred to an epoch $x_4^{(i)}$ when $(g_{44}^{1/2})^i = z^i$ was vanishing and the present time x_4^0 to a value of $z_0 \sim 1$.

From the above formula (7) or (7′) and from the known properties of the elliptic integrals follows for the red shift, given by (6): $\frac{\nu - \nu_0}{\nu_0} = \frac{z - z_0}{z_0} \sim z - 1$, (where $z_0 \sim 1$), that z is a non linear function of x_4. Only for restricted intervals of time the relation between Δx_4 and Δz is nearly linear.

The Hubble coefficient for small and small red shifts results $H \sim 3 \cdot 10^{-18}\,\text{sec}^{-1}$ in good agreement with the observations. If $\frac{\lambda - \lambda_0}{\lambda_0} \sim 2$, $\frac{\nu_0}{\nu} \sim 3$, one obtains from (7′) $x_4 - x_4^{(i)} \sim 10^{17}$ sec. ($3 \cdot 10^9$ light years).

The main conclusion is that the choice of x_4 according to (5) represents the " physical time " indicated by atomic clocks.

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[3] A. WATAGHIN, « Nuovo Cimento S. X. », *55*, 258 (1968); Many other Authors: M. A. MARKOV, PONTECORVO, SMORODINSKY, REINES, ZELDOVIC, etc. have considered before the role of neutrinos in the evolution of the Universe.
[4] J. B. ZELDOVIC, « Uspechi », T. 95, p. 209, 1968 and Notes quoted there.