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On the cosmological constant in a closed universe

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Fisica. — *On the cosmological constant in a closed universe.* Nota [(*)] di ANGELO AGNESE, presentata dal Socio G. WATAGHIN.

RIASSUNTO. — Si dimostra che il valore $\lambda = 0$, in un modello omogeneo e isotropo di universo chiuso relativistico, non è compatibile con i recenti dati dell'osservazione sperimentale, sia nel caso di un universo di radiazione, sia nel caso di un universo di materia.

In this note we show that the value $\lambda = 0$ in a relativistic closed universe model is not compatible with the available observational data, both in the case of a matter universe and in the case of a radiation universe.

We shall adopt the nomenclature used in the Harrison [1] classification. For a uniform universe, isotropic and homogeneus, we have:

$$\dot{R}^2 = C_\nu R^{2-3\nu} + \frac{1}{3}\lambda R^2 - K \tag{1}$$

$$C_\nu = \frac{8}{3}\pi G \rho R^{3\nu} \tag{2}$$

$$\lambda = -3 H_0^2 [q_0 - (3\nu - 2)\sigma_0] \tag{3}$$

$$\sigma_0 = \frac{4\pi G\rho_0}{3 H_0^2} \tag{4}$$

$$H_0 = \left(\frac{\dot{R}}{R}\right)_0 \tag{5}$$

$$q_0 = \left(-\frac{\ddot{R}}{RH^2}\right)_0 \tag{6}$$

where:

$$\begin{cases} \nu = 1 \\ K = 1 \end{cases} \tag{7}$$

for a closed universe of matter;

$$\begin{cases} \nu = \frac{4}{3} \\ K = 1 \end{cases} \tag{8}$$

for a closed universe of radiation.

The available observational data used in this work are:

$$\begin{cases} H_0 = 3.2 \cdot 10^{-18}\ \text{sec}^{-1} & \text{(Hubble constant)} \\ 1.1 < q_0 < 2.5 & \text{(deceleration parameter)} \end{cases} \tag{9}$$

recently given by Allan Sandage [2].

(*) Pervenuta all'Accademia il 19 agosto 1968.

MATTER CASE.

Integrating (1) for $\lambda = 0$, $\nu = 1$, $K = 1$, we have (see ref. [1]):

$$O_1(1, 0, 1) \begin{cases} R = C_1 \sin^2 \chi & (10) \\ t = C_1 (\chi - \sin \chi \cos \chi) & (11) \end{cases}$$

with:

$$C_1 = \frac{8}{3} \pi G \rho R^3 \tag{12}$$

(oscillating universe).

From (10) and (11), eliminating the parameter χ:

$$t = C_1 \arcsin \sqrt{\frac{R}{C_1}} - \sqrt{R(C_1 - R)} \tag{13}$$

and from (3) and (4), always for $\lambda = 0$, $\nu = 1$:

$$\rho_0 = \frac{3 H_0^2 q_0}{4 \pi G} \quad g \cdot cm^{-3} \tag{14}$$

while from (1), (5) and (12) we have:

$$R_0 = \frac{1}{H_0 \sqrt{2 q_0 - 1}} \quad \sec \tag{15}$$

radius of the universe for the $O_1(1, 0, 1)$ model to-day in time units.

Introducing (9) in (14) and (15) we have:

$$\begin{cases} 4.0 \cdot 10^{-29} < \rho_0 < 9.1 \cdot 10^{-29} & (g \cdot cm^{-3}) \\ 4.7 \cdot 10^{27} < R_0 < 8.6 \cdot 10^{27} & (cm). \end{cases} \tag{16}$$

From (13), taking into account (12), (14) and (15) we obtain the age of the universe to-day:

$$t_0 = \frac{1}{2 q_0 - 1} \left(\frac{2 q_0}{\sqrt{2 q_0 - 1}} \arcsin \sqrt{\frac{2 q_0 - 1}{2 q_0}} - 1 \right) \cdot \frac{1}{H_0} \quad \text{(in seconds)} \tag{17}$$

Introducing (9) in (17) we have:

$$4.3 \cdot 10^9 < t_0 < 4.7 \cdot 10^9 \quad \text{(years)} \tag{18}$$

RADIATION CASE.

Integrating (1) for $\lambda = 0$, $\nu = \frac{4}{3}$, $K = 1$, we have (see ref. [1]):

$$O_1\left(1, 0, \frac{4}{3}\right) \begin{cases} R = C_{4/3}^{1/2} \sin \chi & (19) \\ t = C_{4/3}^{1/2} (1 - \cos \chi) & (20) \end{cases}$$

with:

$$C_{4/3} = \frac{8}{3}\pi G \rho R^4 \quad \text{(oscillating universe)}. \tag{21}$$

From (19) and (20), eliminating the parameter χ:

$$t = \sqrt{C_{4/3}} - \sqrt{C_{4/3} - R^2} \tag{22}$$

and from (3) and (4), always for $\lambda = 0$, $\nu = \frac{4}{3}$:

$$\rho_0 = \frac{3 H_0^2 q_0}{8\pi G} \quad g \cdot cm^{-3} \tag{23}$$

while from (1), (5) and (21) we have:

$$R_0 = \frac{1}{H_0 \sqrt{q_0 - 1}} \quad \sec \tag{24}$$

radius of the universe for the $O_1\left(1, 0, \frac{4}{3}\right)$ model to-day in time units.

Introducing (9) in (23) and (24) we have:

$$\begin{cases} 2.0 \cdot 10^{-29} < \rho_0 < 4.6 \cdot 10^{-29} & (g \cdot cm^{-3}) \\ 3.0 \cdot 10^{28} < R_0 < 7.7 \cdot 10^{28} & (cm). \end{cases} \tag{25}$$

From (22), taking into account (21), (23) and (24) we obtain the age of the universe to-day:

$$t_0 = \frac{1}{(\sqrt{q_0} + 1) H_0} \quad \text{(in seconds)} \tag{26}$$

Introducing (9) in (26) we have:

$$3.8 \cdot 10^9 < t_0 < 4.8 \cdot 10^9 \quad \text{(years)} \tag{27}$$

Discussion.

We give below a short summary of observational data:

$$\begin{cases} \rho_0 = 4 \cdot 10^{-30} \, g \cdot cm^{-3} & \text{(for matter) [3]} \\ \rho_0 = 6.8 \cdot 10^{-36} \, g \cdot cm^{-3} & \text{(for radiation) [4-5]} \end{cases} \tag{28}$$

$$R_0 > 4.2 \cdot 10^{27} \quad \text{cm (ref. [3])} \tag{29}$$

$$7 \cdot 10^9 < t_0 < 10^{10} \quad \text{(years) (ref. [6-7])}. \tag{30}$$

For the matter case, the results of our calculation (16) and (18) compared with (28), (29) and (30), indicate that:

a) The radius calculated (always for $\lambda = 0$) would be tolerable.

b) The density is one order of magnitude too great. This happens also in the case $\lambda \neq 0$ (as well known). This discrepancy may be removed if in the future more inter-galactic matter is observed with refined techniques.

c) The calculated universe age is too small, and that seems to make impossible the validity of a $\lambda = 0$ matter model of the universe (whatever be the density).

In the radiation case, the comparison of (25) and (27) with (28), (29) and (30) indicates that:

a) The calculated radius (in a $\lambda = 0$ model) is compatible with the (29) inequality.

b) The calculated density is greater than the photonic observed density (from the 3^0 K observed black body radiation), but could be consistent with observation if a hitherto undetected neutrino black body radiation will in the future be detected (see ref. [8]).

c) The calculated universe age is too small, and that seems to make impossible the validity of a $\lambda = 0$ radiation model of the universe (whatever be the density).

Summarizing we may say that the $\lambda = 0$ case (both in the matter and radiation versions), seems to be inconsistent with observational data on universe age (see point *c*)).

The universe age seems to be compatible with closed models with $\lambda \neq 0$. The mathematical solution and a physical discussion for these models may be found in [9, 10] (for the matter case) and in A. Wataghin [8] (for the radiation case). The latter mathematical solution is also included in the recent Harrison [1] classification.

REFERENCES.

[1] HARRISON E. R., «Monthly Notices», *137*, 69 (1967).
[2] SANDAGE A., Paper at 1967 (January) Texas Symposium on Relativistic Astrophysics in N. Y.
[3] LANDOLT – BORNSTEIN, *VI, I* pag. 675, 676 ed. Springer-Verlag (1965).
[4] PENZIAS A. A. and WILSON R. W., «Ap. J.», *142* (*I*), 419 (1965).
[5] FELTEN J. E., «Phys. Rev. Lett.», *15*, 1003 (1965).
[6] DICKE R. H., «Nature», *194*, 329 (1962).
[7] DEMERQUE R. R. and LARSON R. B., «Ap. J.», *140*, 544 (1964).
[8] WATAGHIN A., «Il N. Cimento», *50 B*, 258 (1968).
[9] LEMAÎTRE G., «Ann. Soc. Sci. Bruxelles», *47 A*, 49 (1927).
[10] FRIEDMAN A., «ZS. f. Phys.», *10*, 337 (1922).