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Low Energy Neutrino Muons at Great Depths

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Fisica. — *Low Energy Neutrino Muons at Great Depths.* Nota di LAURA BERGAMASCO, BENEDETTO D'ETTORRE PIAZZOLI e ANGELO PIANO, presentata (*) dal Socio G. WATAGHIN.

RIASSUNTO. — Si discutono i risultati sui muoni di bassa energia a grandi profondità sottoterra, con particolare riferimento alla componente generata nelle reazioni da neutrino in roccia. Viene calcolato dapprima lo spettro dei neutrini atmosferici al l.d.m., e quindi il rapporto fra il flusso dei μ_ν con energia $<$ 160 MeV (taglio strumentale) ed il flusso totale in funzione della sezione d'urto ν–N usata. Il risultato dimostra che il contributo della componente neutrinica pur crescendo d'importanza con la profondità, resta tuttavia inferiore al risultato sperimentale ottenuto a 4300 m.w.e.

I. INTRODUCTION

The muon intensity underground is due to two components: the first derives from the pion and kaon decay in atmosphere (μ_a) and the second from the neutrino interactions in rock (μ_ν). The relative percentage of the two components is a function of depth x: that is the atmospheric muon flux is initially much higher and then falls rapidly (from 60 m.w.e. to 4300 m.w.e. by a factor $\sim 10^{-5}$) while the neutrino muon flux may be considered almost constant for $x > 1500$ m.w.e. Insofar as the range of ultra high energies will not be attained by great accelerating machines still for many years, the only information on the neutrino characteristics and interactions in the Gev region may come from the deep underground measurements. For instance the preliminary neutrino experiments carried out in Kolar Gold Fields [1] and under Mont Blanc [2] set the first lower limit to the mass of the intermediate boson W, $m_w \geq m_k$.

The parameters which allow an experimental distinction between the atmospheric and neutrino muons are the angular distribution and the energy spectrum. Here we are concerned with the last one: the experimental results on the muon energy spectrum obtained in the M. Blanc Station (vertical depth $x = 4300$ m.w.e. standard rock) with a large volume detector filled with liquid scintillator are of peculiar interest for their anomality and their possible intriguing consequences. Namely, the ratio of stopping muons S (energy released ≤ 160 MeV) to traversing muons N is $(1.19 \pm 0.41) \cdot 10^2$, higher by a factor ~ 30 than the predictions of the conventional interactions theory of atmospheric muons [3, 4]. The same measurements carried out at the shallow depths gave results in agreement with theory at 60 m.w.e. and then slightly higher from 100 to 300 m.w.e. [5]. This discrepancy at

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(*) Nella seduta del 18 giugno 1971.

great depths (supported also by other experimental results at 1500 [6] and 8800 m.w.e. [7]) cannot be easily interpreted.

In this Note we explore the neutrino muon channel and notably: 1) evaluate the atmospheric neutrino flux at sea level, 2) discuss the neutrino–nucleon cross section for the muon production in rock, 3) calculate the ratio $R_\nu = S_\nu/N_\nu$ for the neutrino muons and the total ratio

$$R_{a,\nu} = \frac{S_a + S_\nu}{N_a + N_\nu} = \frac{R_a N_a + R_\nu N_\nu}{N_a + N_\nu} . \tag{1}$$

II. SEA LEVEL NEUTRINO SPECTRUM

The neutrino flux at sea level is made up of two components: one has a galactic (or metagalactic) origin and the other comes from the decay in atmosphere of pions, muons and kaons. The primary component is $\sim 10^{-3}$ of the total flux and is therefore negligible.

Let us first consider the rate of muon neutrinos produced in the pion decay $\pi \rightarrow (\mu^+ + \nu_\mu)$. The mechanism of the decay is known, and the neutrino energy spectrum $D_\nu(E_\nu, E_\pi)$ is easily derived from the pion production spectrum $P_\pi(x, E_\pi, \vartheta)$. This however suffers of the uncertainties affecting the primary nucleon spectrum and it is better to refer it to the sea level muon spectrum, now considered well established up to some hundreds GeV [8], (the energy released to the neutrino in the rest frame is $E_\nu = E_\mu (m_\pi^2 - m_\mu^2)/(m_\pi^2 + m_\mu^2)$).

The neutrino source function calculated following Zatsepin and Kuz'min [9] is

$$G_\nu^\pi(x, E_\nu, \vartheta) = \int_{E_\nu/(1-m_\mu^2/m_\pi^2)}^{\infty} [I_\pi(E_\pi)\, \rho(x, \vartheta)]^{-1}\, P_\pi(x, E_\pi, \vartheta)\, D_\nu(E_\nu, E_\pi)\, dE_\pi \tag{2}$$

where I_π is the pion attenuation length in the atmosphere $= 120\ \text{g/cm}^2$ and $\rho(x, \vartheta)$ is the distribution function of the air density in the atmosphere [10]. As the first indications of our experimental results seem to indicate a prevalence of vertical stopping muons ($\vartheta < 45^\circ$) the integration of eq. (2) over all the atmosphere traversed is made only for the vertical flux: for $\vartheta = 90^\circ$, however the intensity is higher by a factor ~ 2.

We next consider the kaon decay products assuming for the ratio of neutral and charged kaons to pions in atmosphere $K/\pi = 0.20$ [11]. The charged kaon decay modes considered are semileptonic and hadronic, and in this last case the neutrinos are not produced directly, but via the secondary pion decay: $K^+ \rightarrow (\mu^+ + \nu_\mu)$, $(\pi^0 + \mu^+ + \nu_\mu)$, $(\pi^+ + \pi^0)$ $(2\pi^+ + \pi^-)$, $(\pi^+ + 2\pi^0)$. The branching ratios ($\beta \simeq 0.60$, 0.03, 0.21, 0.06, 0.02) are affected by the usual form factors ambiguities: however for our purpose they are sufficiently indicative also for the indeterminacy of many other parameters (kaon-pion ratio, primary energy spectrum, and so on). In the neutral kaon decay the only relevant state is the linear combination of K^0 and K, $K^0_S \to (\pi^+, \pi^-)$ with $\beta = 0.68$. The current theory gives the energy spectra of the pion and charged lepton, and the $\pi - l$ angular and energy correlations as a function of the pion energy: these last quantities may be compared with the available sea level experimental data, and can be used to check the the derived neutrino spectra. Moreover there are recently been two direct experimental results on the $\pi - l$ angular correlations (of course at accelerator energies) which confirm the theoretical assumptions [12, 13].

The source function $G^K_\nu(x, E_\nu, \vartheta)$ for the kaon channel is analogous to eq. (2), taking account however of the different decay and absorption probabilities because of the heavier mass (and hence the higher Q value), and of the shorter mean lifetime. It is just for this difference that the flux of kaon induced neutrinos is negligible at low energies and then becomes dominant (at 10^4 GeV the ratio $\nu(K) : \nu(\pi)$ is ~ 8).

Turning to the muon decay channel $\mu^- \to (e^-, \bar{\nu}_e, \nu_\mu)$ and $\mu^+ \to (e^+, \nu_e, \bar{\nu}_\mu)$ we see that the muon life time is longer than the pion and kaon lifetime by a factor 10^2 (a factor 10^4 for the K^0 states) but its contribution is nevertheless remarkable for the long distance travelled in atmosphere and the high energy fraction transferred to the decay neutrinos ($\sim 60\,\%$, being the maximum energy given to the electron $E^{\mathrm{Max}}_e = (m^2_\mu + m^2_e)/2\, m_\mu$). The vertical spectrum derived from the source function $G^\mu_\nu(x, E_\nu, \vartheta)$ falls however rapidly with energy and already at 50 GeV it is lower than those relative to the pion and kaon decay by at least a factor 5.

Summing up the three contributions we obtain the muon neutrino energy spectrum at sea level: this does have an analytical form (the integration has been carried out numerically) and only above 5 GeV can be roughly approximated by two power laws.

$$N(E_\nu)\, dE_\nu \sim 10^{-1}\, E_\nu^{-3.3} \qquad 5 \leq E_\nu \leq 50\ \mathrm{GeV} \tag{3}$$
$$6.10^{-2}\, E_\nu^{-2.9} \qquad 50 \leq E_\nu \leq 10^4\ \mathrm{GeV}.$$

At the lowest energies, $0.01 < E_\nu < 10$ GeV the calculations are affected by the uncertainties on the sea level muon spectrum under 2 GeV (the problem has been recently explored by Wolfendale [14]) luckily the dominant contribution to the underground muon flux comes from neutrinos of energy above some GeV.

III. NEUTRINO INTERACTIONS IN ROCK

The cosmic ray muon neutrinos traversing the earth undergo reactions such as

$$\nu(k_1) + \alpha(k_2) \to \mu^-(k'_1) + \Gamma(k'_2) \quad ; \quad \bar{\nu}(k_1) + \alpha'(k_2) \to \mu^+(k'_1) + \Gamma'(k'_2)$$

where α or α' denotes the target (nuclear states), and Γ or Γ' any complex of strongly interacting particles. The accelerator experiments have shown

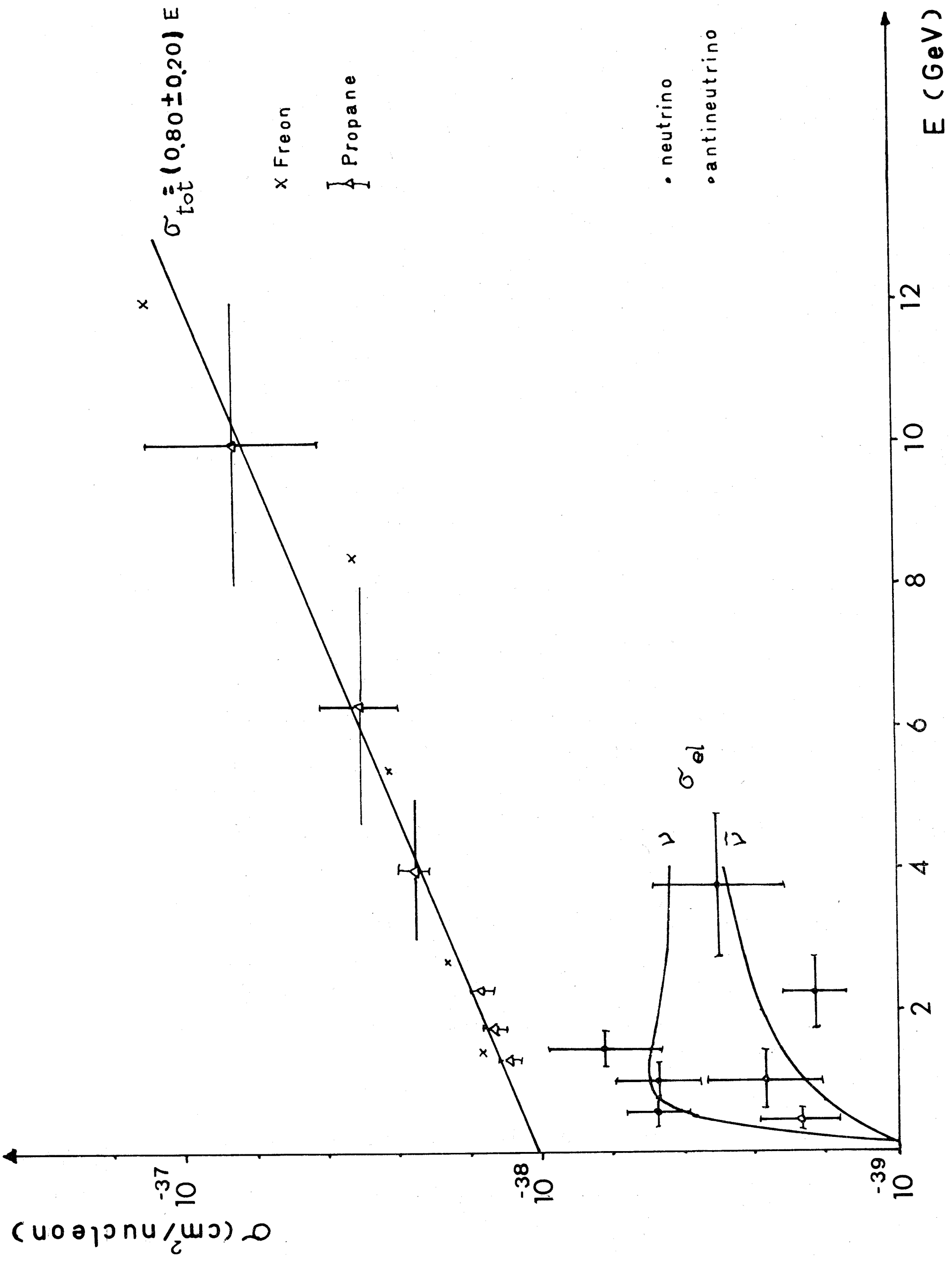
$\sigma_{tot} = (0.80 \pm 0.20)\,E$
× Freon
Propane
• neutrino
∘ antineutrino
E (GeV)
2
4
6
8
10
12
10^{-37}
10^{-38}
10^{-39}
σ (cm^2/nucleon)
σ_{el}
ν
$\bar{\nu}$

Fig. 1.

that at low energies the semileptonic and leptonic processes are satisfactorily described by the (V, A) current-current interaction, but it is rather critical to predict something on the high energy behaviour. Apart from the intermediate vector boson the cross section for elastic processes (in which $\Gamma = p$ and $\Gamma' = n$) is given by

$$(4) \qquad \frac{d\sigma_{el}}{dq^2} = \frac{(G^0)^2}{8\,\pi M^2\,\nu^2}\left[A\,(q^2) \mp B\,(q^2)\,(s-u) + C\,(q^2)\,(s-u)^2\right]$$

where $s = -(k_2 + k_1)^2 = -(k_2' + k_1')^2$ and $u = -(k_2 - k_1')^2 = -(k_2' - k_1)^2$ are the scattering variables, $q^2 = (k_2' - k_2)^2$ is the four-momentum transfer squared, ν is the energy transferred to the nuclear final state, and $A\,(q^2)$, $B\,(q^2)$, $C\,(q^2)$ are structure functions depending on the choice for the axial (F_A) and vector (F_V) form factors. The present experimental data on high energy neutrino reactions do not allow a definite choice between the different types of form factors proposed: however the assumption

$$(5) \qquad F_{A(V)} = (1 + q^2/M^2_{A(V)})$$

$$\text{with} \quad M_V = 0.84\ \text{GeV}/C^2 \qquad \text{and} \quad M_A = 0.8^{+0.13}_{-0.20}\ \text{GeV}/C^2$$

seems to be favoured yielding a cross section practically constant for $E_\nu > 1$ GeV (fig. 1): $\sigma_{el} = (0.7 \pm 0.1)\cdot 10^{-38}$ cm²/nucl.

In the inelastic neutrino interactions the final states Γ or Γ' consist of a nucleon plus one or more mesons. Assuming a point-like leptonic weak current, the differential inelastic cross section is

$$(6) \qquad \frac{d^2\sigma_{in}}{dq^2\,d\nu} = \frac{E_\nu}{EM}\,\frac{G^2}{2\pi}\left[W_2\cos^2\frac{1}{2}\vartheta + \sin^2\frac{1}{2}\vartheta\left(2\,W_1 + \frac{E+E_\mu}{M}\,W_3\right)\right]$$

where W_1, W_2, and W_3 are the dimensionless structure functions depending only on q^2 and ν, and ϑ is the muon production angle relative to the neutrino direction. We shall not get involved in any discussion on the W_i terms, still too much debated, and use instead the latest experimental data obtained with the CERN heavy liquid bubble chamber: from 1 up to 12 GeV they seem to fit a linear variation of the form [15]

$$(7) \qquad \sigma_{tot}\,(E) = (0.80 \pm 0.20)\,E\cdot 10^{-38}\ \text{cm}^2/\text{nucleon}$$

where E is GeV and the error quoted is comprehensive of all the uncertainties (fig. 1). Above 12 GeV there can be only speculations: it is generally thought that the cross section should flatten at some energy value E_0, but this threshold is not known even approximately.

For the antineutrinos the latest accelerator data indicate $\sigma_{\bar\nu} = 0.75 - 1.00\,\sigma_\nu$, and this uncertainty adds to the many others.

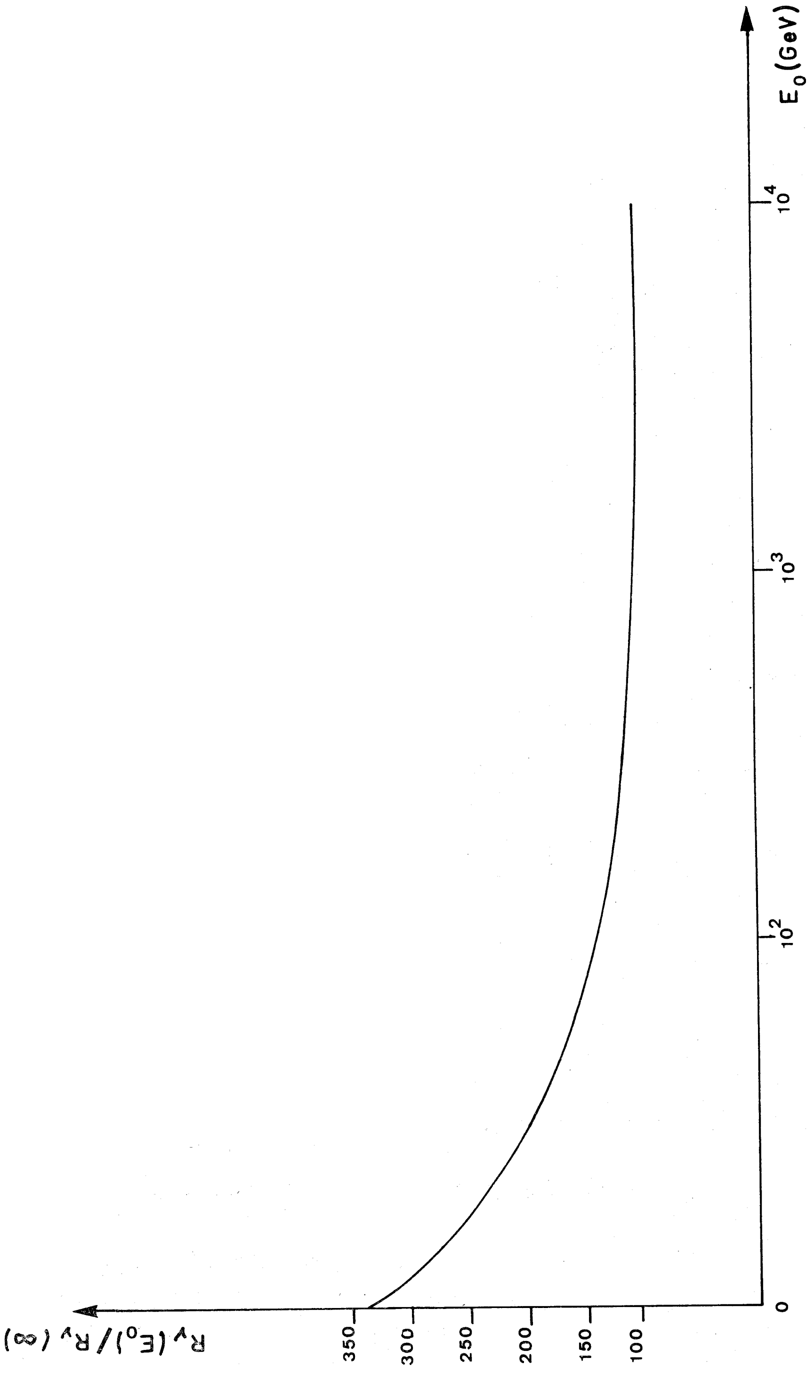

Fig. 2.

IV. NEUTRINO INDUCED MUON SPECTRUM

The differential spectrum $P_\mu(x, E_\mu, \vartheta)$ at depth x of the muons generated in the ν reactions comes from the diffusion equation in rock

$$\frac{\partial P_\mu(x, E_\mu, \vartheta)}{\partial x} = -\frac{\partial}{\partial E}\left[f(E_\mu)\, P_\mu(x, E_\mu, \vartheta)\right] + G_\mu(E_\mu, \vartheta) \tag{9}$$

where $f(E_\mu)$ is the muon energy loss rate ($\mathrm{Mev \cdot gr^{-1} \cdot cm^2}$) and $G_\mu(E_\mu, \vartheta)$ is the generation function, that is the number of muons with energy E_μ created per $\mathrm{gr \cdot cm^{-2}}$. The generation function depends on the interaction cross section of neutrinos and antineutrinos and on the energy transferred to the muon $k_\mu = E_\mu/E_\nu$ [16].

$$G_\mu(E_\mu) = A\mathcal{N}(0.80 \pm 0.20) \begin{cases} E_\mu^{-\gamma} k_\mu^{\gamma-1} & E_\mu \leq k_\mu E_0 \\ E_\mu^{-(\gamma+1)} E_0 k^\gamma & E_\mu > k_\mu E_0 \end{cases} \tag{10}$$

where $\gamma = \gamma(E)$ is the exponent of the neutrino energy spectrum at sea level previously derived.

For $x > 1500$ m.w.e. the state may be considered in equilibrium, that is $\partial P_\mu/\partial x = 0$ and the spectrum

$$P_\mu(E_\mu, \vartheta) = \frac{1}{f(E_\mu)} \int_{E_\mu}^{\infty} G_\mu(E', \vartheta)\, dE' \tag{11}$$

is constant over all great depths. The number of muons generated at depth t that arrive at depth x with a given energy E_μ is then given by

$$P_\mu(E_\mu, t)\, dt\, dE_\mu = G_\mu(E + f(E_\mu)\{x - t\})\, dt\, dE\,. \tag{12}$$

Integration of eq. (12) over all generation depths, from zero down to 4300 m.w.e. gives the differential spectrum of the neutrino induced muons falling vertically on our detector. The ratio of those with energy $E_\mu < 160$ MeV to the total flux is

$$R_\nu = \frac{S_\nu}{N_\nu} = \frac{\displaystyle\int_{0}^{160\,\mathrm{MeV}} P_\mu(E_\mu)\, dE_\mu}{\displaystyle\int_{0\,\mathrm{MeV}}^{\infty} P_\mu(E_\mu)\, dE_\mu}\,. \tag{13}$$

The value of R_ν has been calculated as follows:

1) $f(E_\mu)$: for the many energy loss rates (pair production, bremsstrahlung, nuclear interactions) we used the tabulated functions given is a previous paper of our group [17]. At infinite energies $f(E_\mu = \infty) = 4.55 \cdot 10^{-3} E_\mu$ $\mathrm{MeV \cdot gr^{-1}\, cm^2}$.

2) E_0: the stopping muon rate S_ν is practically independent of the value chosen, but the total rate (integral spectrum) is quite sensitive, and so the ratio drops by a factor ~ 3 passing from $E_0 = 10$ GeV to $E_0 = 10^3$ GeV (fig. 2).

V. CONCLUSIONS

For our purpose we must be sure not to underestimate the ratio R_ν, so we take the highest value allowed by the present experimental data that is $R_\nu (E_0 = 12\ \mathrm{GeV}, E_\mu < 160\ \mathrm{MeV}) = 4.6 \cdot 10^{-2}$. This value is in good agreement with what was obtained by Zatsepin [16], $R_\nu (< 300\ \mathrm{MeV}) \sim 8.10^2$ and Wolfendale [14], $R_\nu (< 70\ \mathrm{MeV}) \sim 1.4 \cdot 10^{-2}$.

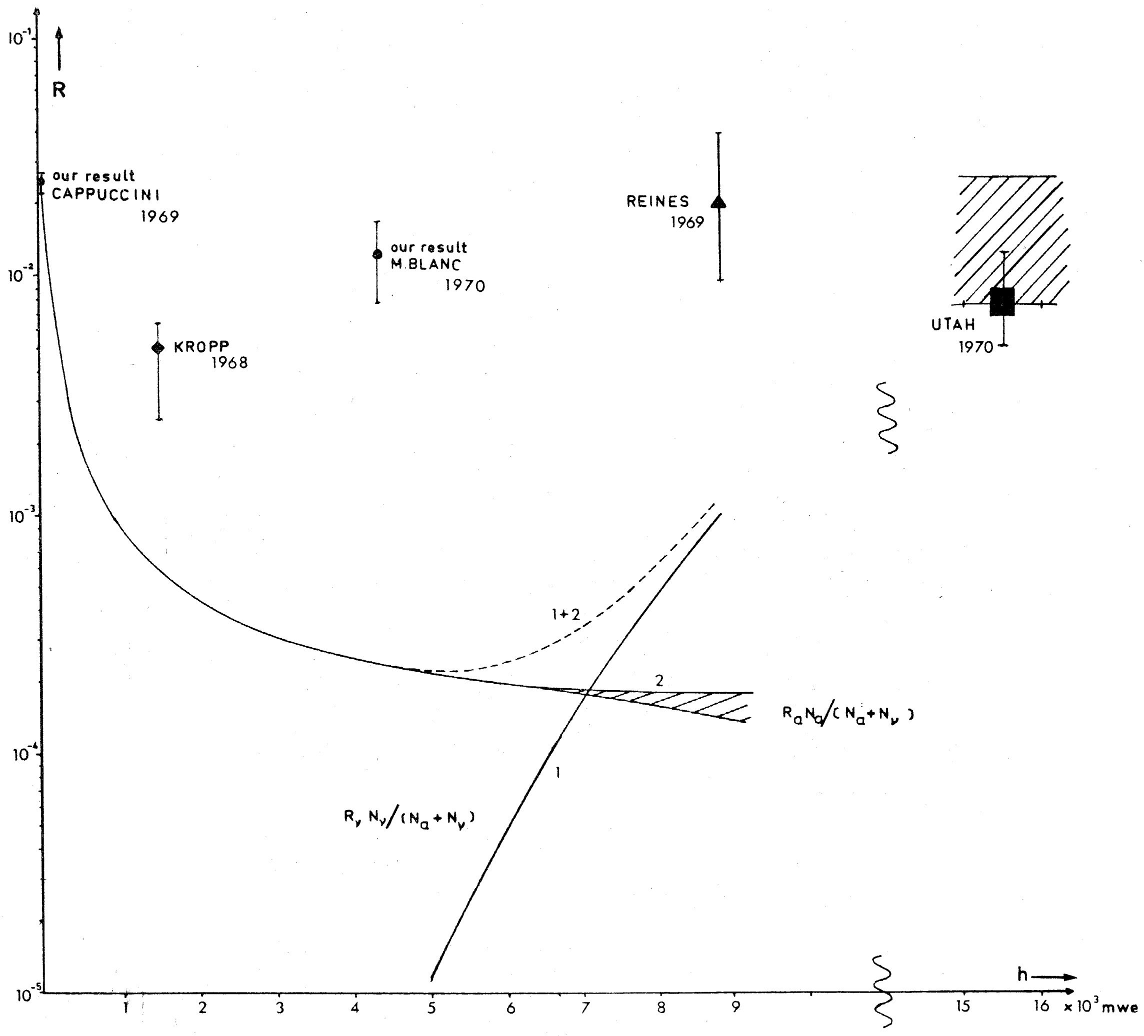

Fig. 3.

The value so derived is constant at all depths, but the importance of its contribute to the measured ratio grows with x, while the flux of atmospheric muons N_a decreases. Fig. 3 shows the two terms $R_\nu N_\nu/(N_\nu + N_a)$ and $R_a N_a/(N_\nu + N_a)$. It may be seen that:

1) at 4300 m.w.e. the atmospheric term is still dominant and the resulting ratio R is clearly lower than the experimental, even taking account of

the straggling; therefore the high value obtained must be given another physical interpretation, perhaps a new leptonic production, process.

2) at ~6000 m.w.e. the two terms are comparable and then the first exceeds the second.

3) at 8800 m.w.e. it is rather difficult to draw a conclusion, even considering that the horizontal flux is about twice the vertical. The stopping muons could actually be neutrino induced, as suggested by Wolfendale [14] and the discrepancy be due to the low statistics, but nothing definite can be asserted.

4) at 15,000 m.w.e. there is a result by the Utah group [18], one muon event of energy between 4 and 10 GeV stopping in the detector. At this depth all muons are surely generated in neutrino interactions and the agreement, even with the many uncertainties, is satisfactory.

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