Atti Accademia Nazionale dei Lincei

Classe Scienze Fisiche Matematiche Naturali

Rendiconti

K. B. Lal, S. S. Singh

On Kählerian spaces with recurrent Bochner curvature

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **51** (1971), n.3-4, p. 213–220.

Accademia Nazionale dei Lincei

<http://www.bdim.eu/item?id=RLINA_1971_8_51_3-4_213_0>

L'utilizzo e la stampa di questo documento digitale è consentito liberamente per motivi di ricerca e studio. Non è consentito l'utilizzo dello stesso per motivi commerciali. Tutte le copie di questo documento devono riportare questo avvertimento.

Articolo digitalizzato nel quadro del programma
bdim (Biblioteca Digitale Italiana di Matematica)
SIMAI & UMI
http://www.bdim.eu/

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Accademia Nazionale dei Lincei, 1971.

Geometria differenziale. — *On Kählerian spaces with recurrent Bochner curvature.* Nota (*) di K. B. LAL e S. S. SINGH, presentata dal Socio E. BOMPIANI.

RIASSUNTO. — Gli spazi riemanniani n–dimensionali di curvatura ricorrente furono introdotti e sviluppati da Ruse [2] (1) e Walker [4] e poi furono studiati da vari altri Autori. Mathai [1] ha definito gli spazi kähleriani simmetrici, ricorrenti e semi–ricorrenti ed ha ottenuto alcuni rapporti esistenti tra di essi. In questa Nota gli Autori hanno definito uno spazio kähleriano con curvatura bochneriana ricorrente, derivandone alcune proprietà relative a spazi kähleriani ricorrenti, semi–ricorrenti e simmetrici.

1. INTRODUCTION

An n $(= 2m)$ dimensional Kählerian space is a Riemannian space if it admits a structure tensor $\varphi_\mu{}^\lambda$ satisfying

(1.1) (2) $$\varphi_\mu{}^\alpha \varphi_\alpha{}^\lambda = - \delta_\mu{}^\lambda$$

where

$$\delta_\mu{}^\lambda = \begin{cases} 1 & \lambda = \mu \\ 0 & \lambda \neq \mu . \end{cases}$$

(1.2) $$\varphi_{\lambda\mu} = - \varphi_{\mu\lambda} \quad , \quad (\varphi_{\lambda\mu} = \varphi_\lambda{}^\alpha g_{\alpha\mu}) ,$$

and

(1.3) $$\varphi_\lambda{}^k{}_{,\mu} = 0$$

where a comma followed by an index denotes the operator of covariant differentiation with respect to the metric tensor g_{ij} of the Riemannian space.

From (1.2) in view of (1.3) we get

(1.4) $$\varphi_{k\lambda,\mu} = 0 .$$

The operators O_{ir}^{sh} and $^*O_{ir}^{sh}$ have been defined in [6] page 133 as

(1.5) $$O_{ir}^{sh} = \frac{1}{2} (\delta_i^s \delta_r^h - \varphi_i^s \varphi_r^h) \quad , \quad {}^*O_{ir}^{sh} = \frac{1}{2} (\delta_i^s \delta_r^h + \varphi_i^s \varphi_r^h)$$

(*) Pervenuta all'Accademia il 15 ottobre 1971.

(1) The numbers in the square brackets refer to the References in the end.

(2) All Latin and Greek indices run over the same range from 1 to n.

which satisfy the relations

$$(1.6) \qquad O + {}^*O = A \ , \quad O \cdot O = O \ , \quad O \cdot {}^*O = 0 \, ,$$
$$ {}^*O \cdot O = 0 \ , \quad {}^*O \cdot {}^*O = {}^*O$$

A being the identity operator.

If a tensor $T^{\cdots r \cdots}_{\cdots s \cdots}$ satisfies

$$O^{sh}_{ir} T^{\cdots r \cdots}_{\cdots s \cdots} = 0 \, ,$$

we say that $T^{\cdots r \cdots}_{\cdots s \cdots}$ is hybrid in r and s and if it satisfies

$${}^*O^{sh}_{ir} T^{\cdots r \cdots}_{\cdots s \cdots} = 0 \, ,$$

we say that $T^{\cdots r \cdots}_{\cdots s \cdots}$ is pure in r and s.

Also if a tensor $T_{\cdots t \cdots s \cdots}$ satisfies

$$O^{ts}_{ji} T_{\cdots t \cdots s \cdots} = 0 \, ,$$

we say that $T_{\cdots t \cdots s \cdots}$ is hybrid in t and s and if it satisfies

$${}^*O^{ts}_{ji} T_{\cdots t \cdots s \cdots} = 0 \, ,$$

we say that it is pure in t and s.

It has been verified in [6] page 63, 68 that the metric tensor g_{ij} and the Ricci tensor denoted by R_{ij} are hybrid in i and j. Hence we get in view of (1.5) and the above definition

$$(1.7) \qquad g_{ij} = g_{sr} \varphi_i^s \varphi_j^r ,$$

$$(1.8) \qquad R_{ij} = R_{sr} \varphi_i^s \varphi_j^r .$$

The Riemannian curvature tensor which we denote by $R_{\lambda\mu\nu}{}^k$ is given by

$$(1.9)^{(3)} \qquad R_{\lambda\mu\nu}{}^k = \partial_\lambda \left\{ {k \atop \mu\nu} \right\} - \partial_\mu \left\{ {k \atop \lambda\nu} \right\} + \left\{ {k \atop \lambda\alpha} \right\} \left\{ {\alpha \atop \mu\nu} \right\} - \left\{ {k \atop \mu\alpha} \right\} \left\{ {\alpha \atop \lambda\nu} \right\} ,$$

whereas the Ricci tensor and the scalar curvature are respectively given by $R_{\mu\nu} = R_{\alpha\mu\nu}{}^\alpha$ and $R = g^{\lambda\mu} R_{\lambda\mu}$.

If we define a tensor $S_{\mu\nu}$ by

$$(1.10) \qquad S_{\mu\nu} = \varphi_\mu{}^\alpha R_{\alpha\nu} ,$$

then we have

$$(1.11) \qquad S_{\mu\nu} = - S_{\nu\mu} .$$

(3) $\partial_\lambda = \partial / \partial x^\lambda$.

Recently S. Tachibana [3] has shown that with respect to real coordinates a tensor $K_{\lambda\mu\nu\omega} = K_{\lambda\mu\nu}{}^{k} g_{k\omega}$ defined by

$$(1.12) \quad K_{\lambda\mu\nu}{}^{k} = R_{\lambda\mu\nu}{}^{k} + \frac{1}{n+4}\{R_{\lambda\nu}\,\delta_\mu^k - R_{\mu\nu}\,\delta_\lambda^k + g_{\lambda\nu}\,R_\mu{}^k - g_{\mu\nu}\,R_\lambda{}^k + S_{\lambda\nu}\,\varphi_\mu{}^k - S_{\mu\nu}\,\varphi_\lambda{}^k + \varphi_{\lambda\nu}\,S_\mu{}^k - \varphi_{\mu\nu}\,S_\lambda{}^k + 2\,S_{\lambda\mu}\,\varphi_\nu{}^k + 2\,\varphi_{\lambda\mu}\,S_\nu{}^k\} - \frac{R}{(n+2)(n+4)}\{g_{\lambda\nu}\,\delta_\mu^k - g_{\mu\nu}\,\delta_\lambda^k + \varphi_{\lambda\nu}\,\varphi_\mu{}^k - \varphi_{\mu\nu}\,\varphi_\lambda{}^k + 2\,\varphi_{\lambda\mu}\,\varphi_\nu{}^k\},$$

has components of the tensor given by S. Bochner [5] (4) and has called this tensor the Bochner curvature tensor.

2. KÄHLERIAN RECURRENT SPACES

DEFINITION 2.1. A Kähler space is said to be recurrent if we have

$$(2.1) \quad R_{\lambda\mu\nu}{}^{k}{}_{,\varepsilon} = a_\varepsilon\, R_{\lambda\mu\nu}{}^{k}$$

for some non-zero vector a_ε, and is called semi-recurrent if it satisfies

$$(2.2) \quad R_{\lambda\mu,\varepsilon} = R_{\lambda\mu}\cdot a_\varepsilon .$$

Multiplying this equation by $g^{\lambda\mu}$ we get

$$(2.2)' \quad R_{,\varepsilon} = R\cdot a_\varepsilon .$$

Remark 2.1. From (2.1) it follows that every Kählerian recurrent space is Kählerian semi-recurrent but the converse is not necessarily true.

DEFINITION 2.2. A Kähler space in which the Bochner curvature tensor satisfies the relation

$$(2.3) \quad K_{\lambda\mu\nu}{}^{k}{}_{,\varepsilon} = a_\varepsilon\, K_{\lambda\mu\nu}{}^{k}$$

for some non-zero vector a_ε, shall be called a Kähler space with recurrent Bochner curvature.

Now if we put

$$(2.4) \quad L_{\lambda\mu} = R_{\lambda\mu} - \frac{R}{2(n+2)}\, g_{\lambda\mu},$$

and

$$(2.5) \quad M_{\lambda\mu} = \varphi_\lambda{}^\alpha\, L_{\alpha\mu} = S_{\lambda\mu} - \frac{R}{2(n+2)}\,\varphi_{\lambda\mu},$$

(4) S. BOCHNER [7], K. YANO and S. BOCHNER [5], page 162.

$K_{\lambda\mu\nu}{}^{k}$ has the following form:

$$(2.6) \qquad K_{\lambda\mu\nu}{}^{k} = R_{\lambda\mu\nu}{}^{k} + \frac{1}{n+4}\left[L_{\lambda\nu}\,\delta_{\mu}{}^{k} - L_{\mu\nu}\,\delta_{\lambda}{}^{k} + g_{\lambda\nu}\,L_{\mu}{}^{k} - M_{\lambda\nu}\,\varphi_{\mu}{}^{k} - g_{\mu\nu}\,L_{\lambda}{}^{k} - M_{\mu\nu}\,\varphi_{\lambda}{}^{k} + \varphi_{\lambda\nu}\,M_{\mu}{}^{k} - \varphi_{\mu\nu}\,M_{\lambda}{}^{k} + 2\,M_{\lambda\mu}\,\varphi_{\nu}{}^{k} + 2\,\varphi_{\lambda\mu}\,M_{\nu}{}^{k}\right].$$

Multiplying (2.6) by $g_{k\omega}$ we get

$$(2.7) \qquad K_{\lambda\mu\nu\omega} = R_{\lambda\mu\nu\omega} + \frac{1}{n+4}\left[L_{\lambda\nu}\,g_{\mu\omega} - L_{\mu\nu}\,g_{\lambda\omega} + g_{\lambda\nu}\,L_{\mu\omega} - g_{\mu\nu}\,L_{\lambda\omega} + M_{\lambda\nu}\,\varphi_{\mu\omega} - M_{\mu\nu}\,\varphi_{\lambda\omega} + \varphi_{\lambda\nu}\,M_{\mu\omega} - \varphi_{\mu\nu}\,M_{\lambda\omega} + 2\,M_{\lambda\mu}\,\varphi_{\nu\omega} + 2\,\varphi_{\lambda\mu}\,M_{\nu\omega}\right]$$

which on differentiation with respect to x^{ε} becomes

$$(2.7)' \qquad K_{\lambda\mu\nu\omega,\varepsilon} = R_{\lambda\mu\nu\omega,\varepsilon} + \frac{1}{n+4}\left[g_{\mu\omega}\,L_{\lambda\nu,\varepsilon} - g_{\lambda\omega}\,L_{\mu\nu,\varepsilon} + g_{\lambda\nu}\,L_{\mu\omega,\varepsilon} - g_{\mu\nu}\,L_{\lambda\omega,\varepsilon} + \varphi_{\mu\omega}\,M_{\lambda\nu,\varepsilon} - \varphi_{\lambda\omega}\,M_{\mu\nu,\varepsilon} + \varphi_{\lambda\nu}\,M_{\mu\omega,\varepsilon} - \varphi_{\mu\nu}\,M_{\lambda\omega,\varepsilon} + 2\,\varphi_{\nu\omega}\,M_{\lambda\mu,\varepsilon} + 2\,\varphi_{\lambda\mu}\,M_{\nu\omega,\varepsilon}\right].$$

Multiplying (2.7) by a_{ε} and subtracting from (2.7)′ we have

$$(2.8) \qquad K_{\lambda\mu\nu\omega,\varepsilon} - a_{\varepsilon}\,K_{\lambda\mu\nu\omega} = R_{\lambda\mu\nu\omega,\varepsilon} - a_{\varepsilon}\,R_{\lambda\mu\nu\omega} + \frac{1}{n+4}\left[g_{\mu\omega}\,(L_{\lambda\nu,\varepsilon} - a_{\varepsilon}\,L_{\lambda\nu}) - g_{\lambda\omega}\,(L_{\mu\nu,\varepsilon} - a_{\varepsilon}\,L_{\mu\nu}) + g_{\lambda\nu}\,(L_{\mu\omega,\varepsilon} - a_{\varepsilon}\,L_{\mu\omega}) - g_{\mu\nu}\,(L_{\lambda\omega,\varepsilon} - a_{\varepsilon}\,L_{\lambda\omega}) + \varphi_{\mu\omega}\,(M_{\lambda\nu,\varepsilon} - a_{\varepsilon}\,M_{\lambda\nu}) - \varphi_{\lambda\omega}\,(M_{\mu\nu,\varepsilon} - a_{\varepsilon}\,M_{\mu\nu}) + \varphi_{\lambda\nu}\,(M_{\mu\omega,\varepsilon} - a_{\varepsilon}\,M_{\mu\omega}) - \varphi_{\mu\nu}\,(M_{\lambda\omega,\varepsilon} - a_{\varepsilon}\,M_{\lambda\omega}) + 2\,\varphi_{\nu\omega}\,(M_{\lambda\mu,\varepsilon} - a_{\varepsilon}\,M_{\lambda\mu}) + 2\,\varphi_{\lambda\mu}\,(M_{\nu\omega,\varepsilon} - a_{\varepsilon}\,M_{\nu\omega})\right].$$

If the Kähler space is Kählerian semi-recurrent then in view of equations (2.2)′, (2.4) and (2.5) we get

$$(2.9) \qquad L_{\lambda\mu,\varepsilon} - a_{\varepsilon}\,L_{\lambda\mu} = 0 \qquad \text{and} \qquad (2.10) \qquad M_{\lambda\mu,\varepsilon} - a_{\varepsilon}\,M_{\lambda\mu} = 0$$

and so the equation (2.8) reduces to

$$(2.11) \qquad K_{\lambda\mu\nu\omega,\varepsilon} - a_{\varepsilon}\,K_{\lambda\mu\nu\omega} = R_{\lambda\mu\nu\omega,\varepsilon} - a_{\varepsilon}\,R_{\lambda\mu\nu\omega}.$$

Conversely, if in a Kähler space the equation (2.11) is satisfied then we have from (2.8) the relation

$$(2.12)\qquad \begin{aligned} & g_{\mu\omega}(L_{\lambda\nu,\varepsilon} - a_\varepsilon L_{\lambda\nu}) - g_{\lambda\omega}(L_{\mu\nu,\varepsilon} - a_\varepsilon L_{\mu\nu}) + \\ & + g_{\lambda\nu}(L_{\mu\omega,\varepsilon} - a_\varepsilon L_{\mu\omega}) - g_{\mu\nu}(L_{\lambda\omega,\varepsilon} - a_\varepsilon L_{\lambda\omega}) + \\ & + \varphi_{\mu\omega}(M_{\lambda\nu,\varepsilon} - a_\varepsilon M_{\lambda\nu}) - \varphi_{\lambda\omega}(M_{\mu\nu,\varepsilon} - a_\varepsilon M_{\mu\nu}) + \\ & + \varphi_{\lambda\nu}(M_{\mu\omega,\varepsilon} - a_\varepsilon M_{\mu\omega}) - \varphi_{\mu\nu}(M_{\lambda\omega,\varepsilon} - a_\varepsilon M_{\lambda\omega}) + \\ & + 2\,\varphi_{\nu\omega}(M_{\lambda\mu,\varepsilon} - a_\varepsilon M_{\lambda\mu}) + 2\,\varphi_{\lambda\mu}(M_{\nu\omega,\varepsilon} - a_\varepsilon M_{\nu\omega}) = 0 . \end{aligned}$$

Multiplying the above equation by $g^{\mu\omega}$ and using the relation (1.2) we get

$$(2.13)\qquad \begin{aligned} & (n-2)(L_{\lambda\nu,\varepsilon} - a_\varepsilon L_{\lambda\nu}) + g_{\lambda\nu}\, g^{\mu\omega}(L_{\mu\omega,\varepsilon} - a_\varepsilon L_{\mu\omega}) - \\ & - \varphi_\lambda{}^\mu (M_{\mu\nu,\varepsilon} - a_\varepsilon M_{\mu\nu}) + \varphi_\nu{}^\omega (M_{\lambda\omega,\varepsilon} - a_\varepsilon M_{\lambda\omega}) + \\ & + 2\,\varphi_\nu{}^\mu (M_{\lambda\mu,\varepsilon} - a_\varepsilon M_{\lambda\mu}) + 2\,\varphi_\lambda{}^\omega (M_{\nu\omega,\varepsilon} - a_\varepsilon M_{\nu\omega}) = 0 . \end{aligned}$$

On using (2.4), (2.5), (1.7) and (1.8) in the above equation we get after some simplification

$$(2.14)\qquad (n+4)(R_{\lambda\nu,\varepsilon} - a_\varepsilon R_{\lambda\nu}) = 0 .$$

Since $n \neq -4$ we have

$$R_{\lambda\nu,\varepsilon} = a_\varepsilon R_{\lambda\nu}$$

which shows that the Kähler space is Kählerian semi-recurrent. We thus have the following

THEOREM 2.1. *The necessary and sufficient condition for a Kähler space to be Kählerian semi-recurrent is*

$$K_{\lambda\mu\nu\omega,\varepsilon} - a_\varepsilon K_{\lambda\mu\nu\omega} = R_{\lambda\mu\nu\omega,\varepsilon} - a_\varepsilon R_{\lambda\mu\nu\omega} .$$

Now if the Bochner curvature tensor vanishes, then from the equation (2.11) we get

$$(2.15)\qquad R_{\lambda\mu\nu\omega,\varepsilon} = a_\varepsilon R_{\lambda\mu\nu\omega} ,$$

which shows that the space is Kählerian recurrent.

We thus have the following Corollary:

COROLLARY. *A Kählerian semi-recurrent space in which the Bochner curvature tensor vanishes is Kählerian recurrent.*

From (2.11) it follows that every Kählerian recurrent space is a Kähler space with recurrent Bochner curvature. Hence we have in view of the above Corollary

THEOREM 2.2. *The necessary and sufficient condition for a Kählerian semi-recurrent space to be Kählerian recurrent is that the space be that of recurrent Bochner curvature.*

From equations (2.8), (2.11) and the Remark 2.1 we have

THEOREM 2.3. *The necessary and sufficient conditions for a Kähler space to be Kählerian recurrent are that the space be that of recurrent Bochner curvature and the equation* (2.12) *be satisfied.*

3. KÄHLERIAN SPACES WITH PARALLEL OR VANISHING BOCHNER CURVATURE TENSOR

DEFINITION 3.1. A Kähler space satisfying the relation

$$K_{\lambda\mu\nu}{}^{k}{}_{,\varepsilon} = 0 \qquad \text{or} \qquad K_{\lambda\mu\nu k,\varepsilon} = 0 \tag{3.1}$$

has been called a Kähler space with parallel or vanishing Bochner curvature tensor.

A Kähler space with recurrent Bochner curvature is characterized by

$$K_{\lambda\mu\nu\omega,\varepsilon} = a_{\varepsilon} K_{\lambda\mu\nu\omega} \tag{3.2}$$

which in view of the equation (3.1) gives

$$a_{\varepsilon} K_{\lambda\mu\nu\omega} = 0$$

or

$$K_{\lambda\mu\nu\omega} = 0 \qquad \text{for} \quad a_{\varepsilon} \neq 0 .$$

Hence we have

THEOREM 3.1. *If a Kähler space with parallel and recurrent Bochner curvature has non-zero recurrence vector then the Bochner curvature tensor vanishes.*

In a Kähler space with parallel Bochner curvature equation (2.8) takes the form

$$\begin{aligned} R_{\lambda\mu\nu\omega,\varepsilon} - a_{\varepsilon} R_{\lambda\mu\nu\omega} + a_{\varepsilon} K_{\lambda\mu\nu\omega} + \frac{1}{n+4} [& g_{\mu\omega} (L_{\lambda\nu,\varepsilon} - a_{\varepsilon} L_{\lambda\nu}) - \\ & - g_{\lambda\omega} (L_{\mu\nu,\varepsilon} - a_{\varepsilon} L_{\mu\nu}) + g_{\lambda\nu} (L_{\mu\omega,\varepsilon} - a_{\varepsilon} L_{\mu\omega}) - \\ & - g_{\mu\nu} (L_{\lambda\omega,\varepsilon} - a_{\varepsilon} L_{\lambda\omega}) + \varphi_{\mu\omega} (M_{\lambda\nu,\varepsilon} - a_{\varepsilon} M_{\lambda\nu}) - \\ & - \varphi_{\lambda\omega} (M_{\mu\nu,\varepsilon} - a_{\varepsilon} M_{\mu\nu}) + \varphi_{\lambda\nu} (M_{\mu\omega,\varepsilon} - a_{\varepsilon} M_{\mu\omega}) - \\ & - \varphi_{\mu\nu} (M_{\lambda\omega,\varepsilon} - a_{\varepsilon} M_{\lambda\omega}) + 2 \varphi_{\nu\omega} (M_{\lambda\mu,\varepsilon} - a_{\varepsilon} M_{\lambda\mu}) + \\ & + 2 \varphi_{\lambda\mu} (M_{\nu\omega,\varepsilon} - a_{\varepsilon} M_{\nu\omega})] = 0 . \end{aligned} \tag{3.3}$$

Now if the Kähler space is Kählerian semi-recurrent then the above equation reduces to

$$R_{\lambda\mu\nu\omega,\varepsilon} - a_{\varepsilon} R_{\lambda\mu\nu\omega} + a_{\varepsilon} K_{\lambda\mu\nu\omega} = 0 . \tag{3.4}$$

Conversely, if in a Kähler space with parallel Bochner curvature the above equation is satisfied then proceeding as in Theorem 2.1 it can be seen that the space is Kählerian semi-recurrent. We thereby have

THEOREM 3.2. *In a Kähler space with parallel Bochner curvature the necessary and sufficient condition for the space to be Kählerian semi-recurrent is that*

$$R_{\lambda\mu\nu\omega,\varepsilon} + a_\varepsilon (K_{\lambda\mu\nu\omega} - R_{\lambda\mu\nu\omega}) = 0 .$$

4. KÄHLERIAN SYMMETRIC SPACE

DEFINITION 4.1. A Kähler space is called Kählerian symmetric in the sense of Cartan if it satisfies

$$R_{\lambda\mu\nu\omega,\varepsilon} = 0 . \tag{4.1}$$

Therefore, clearly a Kählerian symmetric space is Kählerian semi-symmetric.

On using (2.7)′ we have

THEOREM 4.1. *Every Kählerian symmetric space is a Kähler space with parallel Bochner curvature tensor.*

From (2.7)′ it also follows that in a Kählerian symmetric space the Bochner curvature tensor satisfies

$$K_{\lambda\mu\nu\omega,\varepsilon} = 0 . \tag{4.2}$$

Further if the Kählerian symmetric space is Kählerian recurrent also then from (2,1) we have

$$a_\varepsilon R_{\lambda\mu\nu\omega} = 0 . \tag{4.3}$$

Hence for a Kählerian recurrent space with non zero recurrence vector a_ε, in view of the Remark 2.1 and equations (2.11), (4.1), and (4.2) we have

THEOREM 4.2. *For every Kählerian recurrent space which is Kählerian symmetric, the Bochner curvature tensor and the curvature tensor of the space coincide.*

If a Kähler space is flat then its curvature tensor satisfies

$$R_{\lambda\mu\nu}{}^{k} = 0. \tag{4.4}$$

On using (4.4) the equation (1.12) reduces to

$$K_{\lambda\mu\nu}{}^{k} = 0.$$

Hence we obtain

THEOREM 4.3. *In a flat Kähler space the Bochner curvature tensor vanishes.*

It is also known that a Kähler space of constant curvature is flat. We therefore have

THEOREM 4.4. *In a Kähler space of constant curvature the Bochner curvature tensor vanishes.*

REFERENCES

[1] S. MATHAI, *Kählerian recurrent spaces*, « Ganita », *20*, 2 (1969).
[2] H. S. RUSE, *A classification of* K* *spaces*, « Proc. Lond. Math. Soc. », *2*, 53 (1951).
[3] S. TACHIBANA, *On Bochner curvature tensor*, « Nat. Sci. Report Ochanimizu Univ. », *18* (1), 15–19 (1967).
[4] A. G. WALKER, *On Ruse's spaces of recurrent curvature*, « Proc. Lond. Math. Soc. », 2nd series, *52* (1950).
[5] K. YANO and S. BOCHNER, *Curvature and Betti numbers*, « Ann. Math. Stud. », *32* (1953).
[6] K. YANO, *Differential geometry on Complex and almost Complex spaces*, Pergamon Press (1965).
[7] S. BOCHNER, *Curvature and Betti numbers*, II, « Ann. Math. », *50*, 77–93 (1949).