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Commutation formulae in conformal Finsler space II

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Geometria differenziale. — *Commutation formulae in conformal Finsler space II.* Nota [(*)] di K. B. LAL e S. S. SINGH, presentata dal Socio E. BOMPIANI.

RIASSUNTO. — In lavori precedenti [2] [(1)], [3] gli AA. hanno preso in considerazione l'effetto della trasformazione conforme su diverse formule di commutazione per diverse entità geometriche in uno spazio finsleriano. In questo lavoro gli AA. hanno studiato tale effetto su diverse formule di commutazione che comportano il derivato di covariante del primo tipo di Cartan per entità geometriche diverse nello spazio. I risultati ottenuti in questo modo vengono denominati formule di commutazione nello spazio di Finsler conforme.

1. CONFORMAL FINSLER SPACE

Let two distinct metric functions $F(x, \dot{x})$ and $\bar{F}(x, \dot{x})$ be defined over an n–dimensional space F_n which satisfy the requisite conditions for a Finsler space. The two metrics resulting from them are conformal if the corresponding metric tensors $g_{ij}(x, \dot{x})$ and $\bar{g}_{ij}(x, \dot{x})$ are proportional to each other. It has been shown that the factor of proportionality between them is atmost a point function. Thus we have

$$\bar{g}_{ij}(x, \dot{x}) = e^{2\sigma} g_{ij}(x, \dot{x}), \tag{1.1a}$$

$$\bar{g}^{ij}(x, \dot{x}) = e^{-2\sigma} g^{ij}(x, \dot{x}), \tag{1.1b}$$

$$\bar{F}(x, \dot{x}) = e^{\sigma} F(x, \dot{x}), \tag{1.1c}$$

where $\sigma = \sigma(x)$, $g^{ij}(x, \dot{x})$ being the contravariant components of the metric tensor of F_n. The space $\bar{F}_n$ with the entities $\bar{F}, \bar{g}^{ij}$ etc. is called a conformal Finsler space.

We shall use the geometric entities of the conformal Finsler space given by

$$(1.2)a \quad \bar{l}^i(x, \dot{x}) = e^{-\sigma} l^i(x, \dot{x}), \qquad (1.2)b \quad \bar{l}_i(x, \dot{x}) = e^{\sigma} l_i(x, \dot{x}),$$

$$\bar{G}^i(x, \dot{x}) = G^i(x, \dot{x}) - \sigma_m B^{im}, \tag{1.3}$$

$$\bar{G}^i_j(x, \dot{x}) = G^i_j(x, \dot{x}) - \sigma_m \dot{\partial}_j B^{im}, \tag{1.4}$$ [(2)]

$$\bar{G}^i_{jk}(x, \dot{x}) = G^i_{jk}(x, \dot{x}) - \sigma_m \dot{\partial}_k \dot{\partial}_j B^{im}, \tag{1.5}$$

$$\bar{G}^i_{jkl}(x, \dot{x}) = G^i_{jkl}(x, \dot{x}) - \sigma_m \dot{\partial}_l \dot{\partial}_k \dot{\partial}_j B^{im}, \tag{1.6}$$

$$(1.7)a \quad \bar{C}_{ijh}(x, \dot{x}) = e^{2\sigma} C_{ijh}(x, \dot{x}), \qquad (1.7)b \quad \bar{C}^i_{jh}(x, \dot{x}) = C^i_{jh}(x, \dot{x}),$$

(*) Pervenuta all'Accademia il 15 ottobre 1971.

(1) The numbers in the square brackets refer to the References given at the end.

(2) $\partial_i = \partial/\partial x^i$ and $\dot{\partial} = \partial/\partial \dot{x}^i$.

and

(1.8) $$\bar{A}^i_{jk}(x, \dot{x}) = e^\sigma A^i_{jk}(x, \dot{x})$$

where

(1.9)a $\bar{C}_{ijh}(x, \dot{x}) \overset{\text{def}}{=} \dot{\partial}_h \bar{g}_{ij}(x, \dot{x})$, (1.9)b $\bar{C}^l_{ih}(x, \dot{x}) \overset{\text{def}}{=} \bar{g}^{lj}(x, \dot{x}) \bar{C}_{ijh}(x, \dot{x})$,

(1.10)a $B^{hk}(x, \dot{x}) \overset{\text{def}}{=} \frac{1}{2} F^2 g^{hk} - \dot{x}^h \dot{x}^k$, (1.10)b $\sigma_m \overset{\text{def}}{=} \partial_m \sigma$,

(1.10)c $l^i(x, \dot{x}) \overset{\text{def}}{=} \dot{x}^i / F(x, \dot{x})$ and (1.10)d $l_i(x, \dot{x}) \overset{\text{def}}{=} g_{ik} l^k$.

Sinha [4] deduced the relations given by

(1.11) $$\bar{K}^i_{jhk} = K^i_{jhk} + 2 L^i_{j[hk]}$$

and

(1.12) $$\bar{R}^i_{jhk} = R^i_{jhk} + 2 L^i_{j[hk]} + 2 C^i_{jm} L^m_{\nu[hk]} \dot{x}^\nu,$$

where

$$L^i_{jhk} \overset{\text{def}}{=} U^i_{jh|k} + \dot{\partial}_\nu (\Gamma^i_{jh} + U^i_{jh}) \partial_k B^{\nu n} \sigma_n + U^i_{mk} U^m_{hj}$$

and

$$U^h_{ij} \overset{\text{def}}{=} \bar{\Gamma}^h_{ij} - \Gamma^h_{ij}$$

K^i_{jhk}, R^i_{jhk} and $\bar{K}^i_{jhk}$, $\bar{R}^i_{jhk}$ being the Cartan's curvature tensors of F_n and $\bar{F}_n$ respectively whereas Γ^h_{ij}, $\bar{\Gamma}^h_{ij}$ being the Cartan's connection coefficients of the respective spaces as given in [1].

The equation (1.12) can be written as

(1.13) $$\bar{R}^i_{jhk} = R^i_{jhk} + M^i_{jhk}$$

where

$$M^i_{jhk} \equiv 2 L^i_{j[hk]} + 2 C^i_{jm} L^m_{\nu[hk]} \dot{x}^\nu.$$

2. CARTAN'S COVARIANT DERIVATIVES

The covariant derivatives of a vector $X^i(x, \dot{x})$ with respect to x^i in the sense of Cartan are given by Rund [1] as

(2.1) $$X^i/_k(x, \dot{x}) = F \dot{\partial}_k X^i + X^j A^i_{jk},$$

and

(2.2) $$X^i_{/k}(x, \dot{x}) = \partial_k X^i - \dot{\partial}_j X^i G^j_k - X^j \Gamma^{*i}_{jk},$$

where

$$A^i_{jk}(x, \dot{x}) \overset{\text{def}}{=} F(x, \dot{x}) C^i_{jk}(x, \dot{x}) \overset{\text{def}}{=} \frac{1}{2} F g^{ih} \dot{\partial}_k g_{jh}$$

and

$$G^i_{jk}(x, \dot{x})\,\dot{x}^j = \Gamma^{*i}_{jk}(x, \dot{x})\,\dot{x}^j = G^i_k(x, \dot{x})$$

are components of the symmetric tensor and $\Gamma^{*i}_{jk}(x, \dot{x})$ are the Cartan connection coefficients.

It is well known that

$$l^i_{\ |k} = 0 \tag{2.3}$$

and

$$g_{ij|k} = 0\,. \tag{2.4}$$

We have the following commutation formula as given in [1]

$$2\,X^i_{\ /[hk]} = R^i_{jhk}\,X^j - X^i/_j\;K^j_{\nu hk}\,l^\nu .$$

The general form of the commutation formula for a contravariant tensor $T^{j_1\cdots j_q}(x, \dot{x})$ of order q is given by [6] as

$$2\,T^{j_1\cdots j_q}{}_{/[hk]} = -T^{j_1\cdots j_q}/_s\;K^s_{\nu hk}\,l^\nu + \sum_{\alpha=1}^{q} T^{j_1\cdots j_{\alpha-1}\mu j_{\alpha+1}\cdots j_q}\,R^{j_\alpha}_{\mu hk}\,. \tag{2.5}$$

If we denote the Cartan covariant derivatives given by (2.1) with respect to $\bar{g}_{ij}(x, \dot{x})$ by putting a horizontal bar over the same notation of covariant derivative, then we can obtain the following as given in [5]:

$$\bar{l}^i\,\overline{/}_h(x, \dot{x}) = l^i/_h(x, \dot{x})\,, \tag{2.6}$$

$$\bar{l}_i\,\overline{/}_h(x, \dot{x}) = e^{2\sigma}\,l_i/_h(x, \dot{x})\,, \tag{2.7}$$

$$\bar{g}^{ij}\,\overline{/}_h(x, \dot{x}) = e^{-\sigma}\,g^{ij}/_h(x, \dot{x})\,, \tag{2.8}$$

$$\bar{g}_{ij}\,\overline{/}_h(x, \dot{x}) = e^{3\sigma}\,g_{ij}/_h(x, \dot{x})\,, \tag{2.9}$$

$$\bar{C}^k_{im}\,\overline{/}_h(x, \dot{x}) = e^{\sigma}\,C^k_{im}/_h(x, \dot{x})\,, \tag{2.10}$$

and

$$\bar{G}^i\,\overline{/}_j(x, \dot{x}) = e^{\sigma}\,\{G^i/_j - \sigma_m\,(B^{im}/_j - B^{in}\,A^m_{nj})\}\,. \tag{2.11}$$

Further, the covariant derivative of $G^i_j(x, \dot{x})$ with respect to $\dot{x}^n$ in conformal Finsler space is given by

$$\bar{G}^i_j\,\overline{/}_m = \bar{F}\dot{\partial}_m\,\bar{G}^i_j + \bar{G}^\nu_j\,\bar{A}^i_{\nu m} - \bar{G}^i_\nu\,\bar{A}^\nu_{jm}\,.$$

Using the equations (1.1) c, (1.4), (1.8) and (2.1) we get

$$\bar{G}^i_j\,\overline{/}_n = e^{\sigma}\,[G^i_j/_n - \sigma_s\,\{F\dot{\partial}_n\,(\dot{\partial}_j\,B^{is}) + \dot{\partial}_j\,B^{\nu s}\,A^i_{\nu n} - \dot{\partial}_\nu\,B^{is}\,A^\nu_{jn}\}]\,.$$

By adding and subtracting a term $e^{\sigma}\,\dot{\partial}_j\,B^{i\nu}\,A^s_{rn}\,\sigma_s$ and using (2.1) we get

$$\bar{G}^i_j\,\overline{/}_n = e^{\sigma}\,[G^i_j/_n - \sigma_s\,((\dot{\partial}_j\,B^{is})/_n - \dot{\partial}_j\,B^{i\nu}\,A^s_{\nu n})]\,. \tag{2.12}$$

Similarly, we can deduce the relations

(2.13) $$\overline{G}^i_{jm}\,\overline{/}_n = e^{\sigma}[G^i_{jm}\,/_n - \sigma_s\{(\dot\partial_m\,\dot\partial_j\,B^{is})\,/_n - \dot\partial_m\,\dot\partial_j\,B^{i\nu}\,A^s_{\nu n}\}]$$

and

(2.14) $$\overline{G}^i_{jlm}\,\overline{/}_n = e^{\sigma}[G^i_{jlm}\,/_n - \sigma_s\{(\dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{is})\,/_n - \dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{i\nu}\,A^s_{\nu n}\}]\,.$$

3. THE COMMUTATION FORMULAE

THEOREM 3.1. *When* $F_n(x\,,\dot x)$ *and* $\overline{F}_n(x\,,\dot x)$ *are in conformal correspondence, the commutation formulae for the vectors* ($\overline{l}^i$ *and* $\overline{l}_i$) *in the direction of the element of support are given by*

(3.1) *a* $$2\,\overline{l}^i{}_{\overline{/}[hk]} = e^{-\sigma}\,[M^i_{\nu hk} - 2\,l^i\,/_n\,L^n_{\nu[hk]}]\,l^\nu$$

and

(3.1) *b* $$2\,\overline{l}_{i\overline{/}[hk]} = -\,e^{\sigma}\,[M^\nu_{ihk}\,l_\nu + 2\,l_i\,/_n\,L^n_{\nu[hk]}\,l^\nu]\,.$$

Proof. The commutation formula (2.5) in conformal Finsler space for $\overline{l}^i(x\,,\dot x)$ will be given by

(3.2) $$2\,\overline{l}^i{}_{\overline{/}[hk]} = \overline{R}^i_{\nu hk}\,\overline{l}^\nu - \overline{l}^i\,\overline{/}_n\,\overline{K}^n_{\nu hk}\,\overline{l}^\nu.$$

Using equations (1.2) *a*, (1.11), (1.13), (2.6) and (2.3) we get (3.1) *a*.

Again, the commutation formula (2.5) in conformal Finsler space for $\overline{l}_i(x\,,\dot x)$ will be given by

(3.3) $$2\,\overline{l}_{i\overline{/}[hk]} = -\,\overline{l}_\nu\,\overline{R}^\nu_{ihk} - \overline{l}_i\,\overline{/}_n\,\overline{K}^n_{\nu hk}\,\overline{l}^\nu.$$

Using (1.2) *b*, (1.11), (1.13), (2.3) and (2.7) we get (3.1) *b*.

THEOREM 3.2. *When* $F_n(x\,,\dot x)$ *and* $\overline{F}_n(x\,,\dot x)$ *are in conformal correspondence, we have*

(3.2) *a* $$2\,\overline{g}_{ij\overline{/}[ik]} = -\,e^{2\sigma}\,[g_{i\nu}\,M^\nu_{jhk} + g_{\nu j}\,M^\nu_{ihk} + 2\,g_{ij}\,/_n\,L^n_{\nu[hk]}\,l^\nu]\,,$$

(3.2) *b* $$2\,\overline{g}^{ij}{}_{\overline{/}[hk]} = e^{-2\sigma}\,[g^{\nu j}\,M^i_{\nu hk} + g^{i\nu}\,M^j_{\nu hk} - 2\,g^{ij}\,/_n\,L^n_{\nu[hk]}\,l^\nu]$$

and

(3.2) *c* $$2\,\overline{C}^i_{jm\overline{/}[hk]} = [2\,C^i_{jm/[hk]} + C^\nu_{jm}\,M^i_{\nu hk} - C^i_{\nu m}\,M^\nu_{jhk} - C^i_{j\nu}\,M^\nu_{mhk} - 2\,C^i_{jm}\,/_n\,L^n_{\nu[hk]}\,l^\nu]\,.$$

Proof. The pattern of proof of Theorem 3.2 is the same as that of Theorem 3.1.

Similarly, for a conformally invariant tensor of arbitrary rank we have

THEOREM 3.3. *When* $F_n(x, \dot{x})$ *and* $\overline{F}_n(x, \dot{x})$ *are in conformal correspondence, the commutation formulae for the conformally invariant tensor* $\overline{T}^{i_1 \cdots i_p}_{j_1 \cdots j_q}$ *will be given by*

$$(3.3)\qquad 2\,\overline{T}^{i_1\cdots i_p}_{j_1\cdots j_q\,\overline{/}[hk]} = 2\,T^{i_1\cdots i_p}_{j_1\cdots j_q\,/[hk]} + \sum_{\alpha=1}^{p} T^{i_1\cdots i_{\alpha-1}\,\mu\, i_{\alpha+1}\cdots i_p}_{j_1\cdots\cdots\cdots\cdots j_q}\, M^{i_\alpha}_{\mu hk} - $$

$$- \sum_{\beta=1}^{q} T^{i_1\cdots\cdots\cdots\cdots i_p}_{j_1\cdots j_{\beta-1}\,\mu\, j_{\beta+1}\cdots j_q}\, M^{\mu}_{j_\beta hk} - 2\,T^{i_1\cdots i_p}_{j_1\cdots j_q}/_n\, L^n_{\nu[hk]}\, l^\nu .$$

THEOREM 3.4. *When* $F_n(x, \dot{x})$ *and* $\overline{F}_n(x, \dot{x})$ *are in conformal correspondence, the commutation formulae for* $\overline{G}^i(x, \dot{x})$ *and* $\overline{G}^i_j(x, \dot{x})$ *are given by*

$$(3.4)\,a\qquad 2\,\overline{G}^i_{\overline{/}[hk]} = [2\,G^i_{/[hk]} + G^\nu M^i_{\nu hk} - 2\,G^i/_n\, L^n_{\nu[hk]}\, l^\nu -$$

$$- \sigma_s\,\{2\,B^{is}_{/[hk]} - B^{i\nu} R^s_{\nu hk} + B^{\nu s} M^i_{\nu hk} +$$

$$+ B^{it} A^s_{tn} K^n_{\nu hk}\, l^\nu - 2\,L^n_{\nu[hk]}\,(B^{is}/_n - B^{it} A^s_{tn})\, l^\nu\}] .$$

and

$$(3.4)\,b\qquad 2\,\overline{G}^i_{j\overline{/}[hk]} = [2\,G^i_{j/[hk]} + G^\nu_j M^i_{\nu hk} - G^i_\nu M^\nu_{jhk} -$$

$$- 2\,G^i_j/_n\, L^n_{\nu[hk]}\, l^\nu - \sigma_s\,\{2\,(\dot{\partial}_j B^{is})_{/[hk]} -$$

$$- \dot{\partial}_j B^{i\nu} R^s_{\nu hk} + \dot{\partial}_j B^{\nu s} M^i_{\nu hk} - \dot{\partial}_\nu B^{is} M^\nu_{jhk} +$$

$$+ \dot{\partial}_j B^{it} A^s_{tn} K^n_{\nu hk}\, l^\nu - 2\,L^n_{\nu[hk]}\,((\dot{\partial}_j B^{is})/_n - \dot{\partial}_j B^{it} A^s_{tn})\, l^\nu\}] .$$

Proof. The commutation formula (2.5) for $\overline{G}^i(x, \dot{x})$ in conformal Finsler space will be given by

$$(3.5)\qquad 2\,\overline{G}^i_{\overline{/}[hk]} = \overline{R}^i_{\nu hk}\, \overline{G}^\nu - \overline{G}^i\,\overline{/}_n\, \overline{K}^n_{\nu hk}\, \bar{l}^\nu .$$

By using equations (1.2) a, (1.3), (1.11), (1.13), (2.5) and (2.11) the above equation reduces to

$$(3.6)\qquad 2\,\overline{G}^i_{\overline{/}[hk]} = 2\,G^i_{/[hk]} - \sigma_s\,\{B^{\nu s} R^i_{\nu hk} - B^{is}/_n\, K^n_{\nu hk}\, l^\nu\} +$$

$$+ M^i_{\nu hk}\,\{G^\nu - \sigma_s\, B^{\nu s}\} - 2\,L^n_{\nu[hk]}\,\{G^i/_n - \sigma_s\,(B^{is}/_n -$$

$$- B^{it} A^s_{tn}\}\, l^\nu - \sigma_s\, B^{it} A^s_{tn} K^n_{\nu hk}\, l^\nu .$$

Now, by adding and subtracting a term $B^{i\nu} R^s_{\nu hk}$ in the first bracket and using (2.5) once more we get (3.4) a.

Again, the commutation formula (2.5) for $\overline{G}^i_j(x, \dot{x})$ in conformal Finsler space will be given by

$$(3.7)\qquad 2\,\overline{G}^i_{j\overline{/}[hk]} = \overline{R}^i_{\nu hk}\, \overline{G}^\nu_j - \overline{G}^i_\nu\, \overline{R}^\nu_{jhk} - \overline{G}^i_j\,\overline{/}_n\, \overline{K}^n_{\nu hk}\, \bar{l}^\nu$$

which on using equations (1.2) *a*, (1.4), (1.11), (1.13), (2.5) and (2.12) gives

$$(3.8)\qquad 2\,\bar{G}^i_{j\bar{/}[hk]} = 2\,G^i_{j/[hk]} - \sigma_s\,\{\dot\partial_j\,B^{\nu s}\,R^i_{\nu hk} - \dot\partial_\nu\,B^{is}\,R^\nu_{jhk} - (\dot\partial_j\,B^{is})\,/_n\,K^n_{\nu hk}\,l^\nu\} + M^i_{\nu hk}\,\{G^\nu_j - \sigma_s\,\dot\partial_j\,B^{\nu s}\} - M^\nu_{jhk}\,\{G^i_\nu - \sigma_s\,\dot\partial_\nu\,B^{is}\} - 2\,L^n_{\nu[hk]}\,\{G^i_j\,/_n - \sigma_s\,((\dot\partial_j\,B^{is})\,/_n - \dot\partial_j\,B^{it}\,A^s_{tn})\}\,l^\nu - \sigma_s\,\dot\partial_j\,B^{it}\,A^s_{tn}\,K^n_{\nu hk}\,l^\nu .$$

Now, adding and subtracting a term $\dot\partial_j\,B^{i\nu}\,R^s_{\nu hk}$ in the first bracket and using (2.5) once more we get (3.4) *b*.

THEOREM 3.5. *When* $F_n(x\,,\dot x)$ *and* $\bar F_n(x\,,\dot x)$ *are in conformal correspondence, the commutation formulae for* $\bar G^i_{jm}(x\,,\dot x)$ *and* $\bar G^i_{jlm}(x\,,\dot x)$ *will be given by*

$$(3.5)\,a\qquad 2\,\bar G^i_{jm\bar/[hk]} = [2\,G^i_{jm/[hk]} + G^\nu_{jm}\,M^i_{\nu hk} - G^i_{\nu m}\,M^\nu_{jhk} - G^i_{j\nu}\,M^\nu_{mhk} - 2\,G^i_{jm}\,/_n\,L^n_{\nu[hk]}\,l^\nu - \sigma_s\,\{2\,(\dot\partial_m\,\dot\partial_j\,B^{is})_{/[hk]} - \dot\partial_m\,\dot\partial_j\,B^{i\nu}\,R^s_{\nu hk} - \dot\partial_m\,\dot\partial_j\,B^{\nu s}\,M^i_{\nu hk} - \dot\partial_m\,\dot\partial_\nu\,B^{is}\,M^\nu_{jhk} - \dot\partial_\nu\,\dot\partial_j\,B^{is}\,M^\nu_{mhk} + \dot\partial_m\,\dot\partial_j\,B^{it}\,A^s_{tn}\,K^n_{\nu hk}\,l^\nu - 2\,L^n_{\nu[hk]}\,((\dot\partial_m\,\dot\partial_j\,B^{is})\,/_n - \dot\partial_m\,\dot\partial_j\,B^{it}\,A^s_{tn})\,l^\nu\}]$$

and

$$(3.5)\,b\qquad 2\,\bar G^i_{jlm\bar/[hk]} = [2\,G^i_{jlm/[hk]} + G^\nu_{jlm}\,M^i_{\nu hk} - G^i_{\nu lm}\,M^\nu_{jhk} - G^i_{j\nu m}\,M^\nu_{lhk} - G^i_{jl\nu}\,M^\nu_{mhk} - 2\,G^i_{jlm}\,/_n\,L^n_{\nu[hk]}\,l^\nu - \sigma_s\,\{2\,(\dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{is})_{/[hk]} - \dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{i\nu}\,R^s_{\nu hk} + \dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{\nu s}\,M^i_{\nu hk} - \dot\partial_m\,\dot\partial_l\,\dot\partial_\nu\,B^{is}\,M^\nu_{jhk} - \dot\partial_m\,\dot\partial_\nu\,\dot\partial_j\,B^{is}\,M^\nu_{lhk} - \dot\partial_\nu\,\dot\partial_l\,\dot\partial_j\,B^{is}\,M^\nu_{mhk} + \dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{it}\,A^s_{tn}\,K^n_{\nu hk}\,l^\nu - 2\,L^n_{\nu[hk]}\,((\dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{is})\,/_n - \dot\partial_m\,\dot\partial_l\,\dot\partial_j\,B^{it}\,A^s_{tn})\,l^\nu\}] .$$

Proof. The pattern of proof of this Theorem is the same as that of Theorem 3.4.

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