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Compactifications and function algebras

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Topologia. — *Compactifications and function algebras.* Nota di DAVID S. WOODRUFF, presentata (*) dal Socio G. ZAPPA.

RIASSUNTO. — Si costruiscono compattificazioni di uno spazio X usando certe sottigliezze di funzioni su X (le cosiddette algebre di stone complete) che sono valutate in un campo completo separato uniforme k. In tal modo si generalizzano le compattificazioni introdotte da altri autori.

SECTION 1. INTRODUCTION

We shall construct compactifications of a space X using certain subalgebras of functions F, to be called complete stone algebras, on the space X which are valued in a complete separated uniform field k. This construction generalizes compactifications found by Sultan [17] and Bachman, Beckenstein, Narici and Warner [1]. We then find generalizations of the Stone-Cech compactification, the Gelfand-Kolmogoroff Theorem, and Wallman compactifications. We will find that these theories can be fully generalized by using complete stone algebras of functions which have their values in a locally compact field.

The major construction is given in Section 2): if F is a complete stone algebra on a space X valued in a complete uniform field, then there corresponds to F a compactification for X which we denote $\beta_F X$, a generalization of the Stone-Cech compactification. If the field k is locally compact, then F is isomorphic to $C(\beta_F X, k)$, which is the set of all continuous k-valued functions of $\beta_F X$, and we generalize a result found in Gelfand, Raikov and Shilov [6] by finding a one-to-one correspondance between complete stone algebras and compactifications of X. It is further shown that when F is a complete stone algebra over a locally compact field k, then $\beta_F X$ is homeomorphic to the space of k-valued homomorphisms on F, thus displaying a generalized Gelfand-Kolmogoroff Theorem.

In Section 3) it is shown that when a condition of normality is demanded of an algebra F, a Wallman compactification of X homeomorphic to $\beta_F X$ can be constructed using the zerosets of F. This compactification generalizes that found by Gordon [8] for real valued functions, and, since real valued function algebras which are closed under bounded inversion are shown to be normal in our sense, our Wallman construction subsumes the β-like compactifications of Mrowka [12].

More particulars can be found in Woodruff [20].

(*) Nella seduta del 14 maggio 1977.

Knowledge of zerosets, filters, ultraregular and ultranormal spaces, and nonarchimedean fields is assumed. R, C and H will denote respectively the reals, the complex numbers, and the quaternions. Given a set X and a topological field k, C (X, k) will be the algebra of continuous functions on X over k and C* (X, k) will be the subalgebra of C (X, k) comprised of functions with relatively compact range. A subalgebra F of C* (X, k) will be called a *stone algebra* if F contains constant functions, separates points in X (i.e., given $x, y \in X$ such that $x \neq y$, then there exists $f \in F$ such that $f(x) \neq f(y)$), and self-adjoint when k is C or H. Given that k is a uniform space, F is called a *complete stone algebra* if it is a stone algebra which is complete in the uniformity of uniform convergence ([11], p. 226). A space X will be said to have the *weak*-F *topology* if it has the weakest topology for which each function of F is continuous. X will be called *k-completely regular* (*by* F) if there exists a complete stone algebra (specifically F) for X over k for which X has the weak-F topology. $\mathcal{U}$ Will be called a *weak*-F *uniformity* for X if it is the weakest for which all functions in F are uniformly continuous. F will be called *normal* if whenever Z_1 and Z_2 are disjoint zerosets in Z (F), then there exists $f \in F$ such that $f(Z_1) = 0$ and $f(Z_2) = 1$. This is the (generalized) normal condition of Sultan [17], and it is stronger than the normal conditions of Frink [5], Gordan [8] and Wallman.

PROPOSITION 1.1. *If k is a locally compact topological field, then it is* R, C, H *or a complete nonarchimedean valued field.*

Proof. Follows from [10] and [3], 1) VI 9.3, Cor. 2.

PROPOSITION 1.2. *If* T *is compact and k is a locally compact field then*

a) *k is complete and* C (T, k) *is complete in the topology of uniform convergence,*

b) T *has the weak*-C (T, k) *topology,*

c) *If k is archimedean,* C (T, k) *is a complete stone algebra. If k is nonarchimedean,* C (T, k) *is a complete stone algebra iff* T *is ultraregular.*

All of the following hold when in addition C (T, k) *is guarenteed to be a complete stone algebra*:

d) *if* $A, B \subset T$ *are disjoint closed sets, then there exists* $f \in C(T, k)$ *such that* $f(A) = 0$ *and* $f(B) = 1$,

e) (*Stone-Weierstrass Theorem*) C (T, k) *is the unique complete stone algebra for* T *over k.*

f) *the zerosets of* C (T, k) *are a base for the topology of* T,

g) *The zerosets of* C (T, k) *are a base for the neighborhoods of* T,

h) *if k is commutative, the maximal ideals of* C (T, k) *are of the form* $M_t = \{f \in C(T, k) \mid f(t) = 0$ *for some* $t \in T\}$.

SECTION 2. THE $\beta_F X$ COMPACTIFICATION

Let X be a set, k be a T_2 complete topological field, and F be a complete stone algebra over k. Since the range of each $f \in F$ is contained in a relatively compact set, we may write $f_i(X) \subset S_i$ for each $f_i \in F$ where S_i is a compact set in k. There exists a unique separable uniformity for S_i, and we may give X the weak-F uniformity, $\mathscr{U}$, making each $f_i \in F$ uniformly continuous from X to S_i ([3] 2), II, 2.3 Prop. 4). If $(X, \mathscr{U})$ is the uniform space, then X is T_2 since k is T_2 and F separates points. Let the map $\psi : X \to \pi S_i$ be defined as $\psi(x) = \{f_i(x) \mid f_i \in F\}$. Then since X has the weakest uniformity such that each $f_i \in F$ is uniformly continuous, ψ is a uniform isomorphism from $(X, \mathscr{U})$ onto $\psi(X)$ with the relative product topology of πS_i ([3] 2) II, P 9, Prop. 18). Then ψ is a homeomorphism from X to $\psi(X)$ when we give X the weak-F topology. Indeed the following holds as also in [17].

PROPOSITION 2.1. *Let* X *have the weak*-F *topology, then* $\overline{\psi(X)} \subset \pi S_i$ *is a* T_2 *compactification of* X, *and each* $f_i \in F$ *can be extended uniquely to a uniformly continuous function* $\bar{f}_i : \overline{\psi(X)} \to S_i$.

On $\overline{\psi(X)} = \bar{X}$ define an equivalence relation R as follows: if $s, t \in \bar{X}$ then $s \sim t$ iff $\bar{f}(s) = \bar{f}(t)$ for all $f \in F$. Let s' denote the class of elements equivalent to s, and take $\beta_F X = \bar{X}/R$ to be the usual quotient space with quotient topology $\mathcal{O}_\beta$. Let p be the projection map $p : \bar{X} \to \beta_F X$ defined by $p(t) = t'$, then one readily shows that $p : X \to p(X)$ is a homeomorphism, and hereafter we identify X and $p(X)$. Now, for each $f \in F$ define $f^\beta \in F^\beta :$ $: \beta_F X \to k$ by $f^\beta(t') = \bar{f}(t)$ for all $t' \in \beta_F X$ such that $t \in t'$. The following is easily proved:

LEMMA 2.2. *For each* $f \in F$, f^β *is a continuous extension of* f, $f \to f^\beta$ *is an injection, and* F^β *separates points in* $\beta_F X$.

THEOREM 2.3. *Let* F *be a complete stone algebra for* X *over* k, *where* k *is a* T_2 *complete topological field. Then if* X *has the weak*-F *topology*, $\beta_F X$, *is a* T_2 *compactification of* X. *Further, if* k *is locally compact, then* $C(\beta_F X, k) = F^\beta$ *which is isomorphic to* F.

Proof. $\beta_F X$ is certainly compact and T_2. Let V_x be an open neighborhood of any $x \in p(X)$, then $p^{-1}(V_x) \cap X \neq \varnothing$ since X is dense in $\bar{X} = p(X)$. Then $V_x \cap p(X) \neq \varnothing$ for all $x \in \beta_F X$ and each neighborhood V_x of x, showing that X is dense in $\beta_F X$. Now, F^β is a stone subalgebra of $C(\beta_F X, k)$, then it suffices to show that F^β is complete by 1.2. That F^β is complete follows from the fact that F is complete, from $|f^\beta(t) - g^\beta(t)| = |f(t) - g(t)|$, and from the fact that X is dense in $\beta_F X$. Clearly F is isomorphic to F^β.

PROPOSITION 2.4. *Let* T *be a* T_2 *compactification of* X, *and let* k *be a locally compact field such that* $C(T, k)$ *is a complete stone algebra. Letting* F *be the set of restrictions of* $C(T, k)$ *to* X, T *is homeomorphic to* $\beta_F X$.

Proof. T is a uniform space in the weak-C (T , k) topology. If X , $\mathcal{U}$ is the restricted uniform space, then $\mathcal{U}$ is the weak-F uniformity. Since the compactification $\beta_F X$ is a completion of X in $\mathcal{U}$, then the identity map on X extends to a uniform isomorphism of T onto $\beta_F X$ ([3] II, P 3.7, Cor. to Them. 2).

PROPOSITION 2.5. *Let k_1 , k_2 be locally compact fields, and let μ be an isometric isomorphism taking k_1 into k_2. If* F_1 *and* F_2 *are complete stone algebras for* X *over k_1 and k_2 respectively, then $\mu \circ F_1$ is a complete stone algebra and* $\mu \circ F_1 \subset F_2$ *iff* $\beta_{F_1} X \leq \beta_{F_2} X$.

Proof. Clearly $\mu \circ F_1$ is a complete stone algebra. Denote $\beta_{F_1} X$ as $\beta_1 X$ and $\beta_{F_2} X$ as $\beta_2 X$. Suppose $\beta_1 X \leq \beta_2 X$, then there exists φ, continuous, such that φ is the identity on X and $\varphi : \beta_2 X \to \beta_1 X$. Now for each $f \in F_1$, $\mu \circ f^{\beta_1} \circ \varphi = (\mu \circ f)^{\beta_2}$ holds on $\beta_2 X$, for both sides are continuous and agree on X. Thus $\mu \circ f^{\beta_1} \circ \varphi \in C(\beta_2 X , k_2) = F_2$ by 2.3. But when restricted to X this shows that $\mu \circ f \in F_2$ for each $f \in F_1$. Conversely, suppose $\mu \circ F_1 \subset F_2$. Let $(X , \mathcal{U}_1)$ and $(X , \mathcal{U}_2)$ be the respective weak-F_1 and weak-F_2 uniform spaces, and let i be the identity on X such that $i : (X , \mathcal{U}_2) \to (X , \mathcal{U}_1)$. If V_1 is any entourage in the uniform structure of k_1, and $V_2 = (\mu \times \mu) V_1$, then $(f \times f)^{-1} (V_1) = (\mu \circ f \times \mu \circ f)^{-1} (V_2)$. Then each subbasis entourage in $\mathcal{U}_1$ is a member of $\mathcal{U}_2$, and thus $i : (X , \mathcal{U}_2) \to (X , \mathcal{U}_1)$ is uniformly continuous. Then i may be be extended to a uniformly continuous map $\varphi : \beta_2 X \to \beta_1 X$.

Propositions 2.3, 2.4 and 2.5 show that there exists an order preserving one-to-one correspondance between the complete stone algebras over X and the T_2 compactifications of X.

The following generalize readily from [1]. Use is made of the smallest closed subalgebra $C_S (X , k)$ of $C^* (X , k)$ which contains the characteristic functions and $\beta_0 X$, the Banachewski compacification ([1]).

PROPOSITION 2.6. $C_S (X , k) = C^* (X , k)$ *for any nonarchimedean field k when* X *is ultraregular, and for any locally compact field k when* X *is ultranormal.*

PROPOSITION 2.7. *Let* $C^* = C^* (X , k)$. *If* X *is ultraregular, then* $\beta_S = \beta_0 X = \beta_{C^*} X$ *for all nonarchimedean fields k, and if* X *is ultranormal then* $\beta_0 X = \beta_{C^*} X = \beta X$ *for all locally compact fields k.*

One obtains a generalization of the Gelfand-Kolmogoroff theorem as follows; take $\mathcal{H} (F , k) , \mathcal{O}_{\mathcal{H}}$ to be the k-valued homomorphisms on F with the weak-J_f topology, where $J_f : \mathcal{H} \to k$ is defined as $J_f (h) = f (h)$ for all $h \in \mathcal{H}$. Then generalizing [17]:

PROPOSITION 2.8. *If k is a commutative locally compact field, and* F *is a complete stone algebra for* X *over k then* $\beta_F X$ *is homeomorphic to* $\mathcal{H} (F , k)$.

COROLLARY 2.9. *If* T *is compact and k is* R *or* C *then* T *is homeomorphic to* $\mathcal{H} (C (T , k) , k)$. *If* T *is compact and ultraregular, and k is any locally compact field, then* T *is homeororphic to* $\mathcal{H} (C (T , k) , k)$.

SECTION 3. A WALLMAN COMPACTIFICATION

Construction of a Wallman compactification will proceed using the following condition: *Condition N*: Given $F \subset C^*(X, k)$, *a*) X is T_1 and k-completely regular by F where k is locally compact. *b*) The intersection of two zerosets of F is a zeroset. *c*) If Z_1, Z_2 are zerosets, then $\bar{Z}_1 \cap \bar{Z}_2 = \overline{Z_1 \cap Z_2}$, where closure is in $\beta_F X$. *d*) If $Z_1 \cap Z_2 = \varnothing$ then $\bar{Z}_1 \cap \bar{Z}_2 = \varnothing$.

Z-filters are nonempty subfamilies of Z (F), and properties like those of filters are easily proved of them. Let $W(Z, F) = \{\mathscr{F} \mid \mathscr{F}$ is a Z-ultrafilter in $Z(F)\}$, and consider the collection $\{\mathscr{D}(Z) \mid Z \in Z(F)\}$ of all sets $\mathscr{D}(Z) = \{\mathscr{F} \in W(Z, F) \mid Z \in \mathscr{F}\}$. If $\mathscr{F} \in \mathscr{D}(Z_1) \cup \mathscr{D}(Z_2)$, then $Z_1, Z_2 \in \mathscr{F}$. Thus $Z_1 \cap Z_2 \in \mathscr{F}$ since $\mathscr{F}$ is a Z-ultrafilter, and then $\mathscr{F} \in \mathscr{D}(Z_1 \cap Z_2)$. We see then that $\mathscr{D}(Z_1) \cup \mathscr{D}(Z_2) \subset \mathscr{D}(Z_1 \cap Z_2)$, from which it follows that $\{\mathscr{D}(Z) \mid Z \in Z(F)\}$ is a base of closed sets for some topology $\mathscr{W}$ on $W(Z, F)$. We suppose that $W(Z, F)$ is topologized by $\mathscr{W}$. Denote $\{Z \in Z(F) \mid t \in \bar{Z}\}$ by $\mathscr{F}(t)$, where $t \in \beta_F X$ (all closures will be in $\beta_F X$). One can show under Condition N that $\mathscr{F}(t)$ is a Z-ultrafilter. Since $t \in \bar{Z}$ for all $Z \in \mathscr{F}(t)$ then t is an adherence point of $\mathscr{F}(t)$. We define $\theta : \beta_F X \to W(Z, F)$ so that $t \to \mathscr{F}(t)$. Under condition N, if $\mathscr{F}$ is a Z-ultrafilter in X then $\bar{\mathscr{F}} = \{\bar{Z} \mid Z \in \mathscr{F}\}$ is a $\bar{Z}$-ultrafilter. That is, $\bar{\mathscr{F}}$ is an ultrafilter in the collection of sets $\{\bar{Z} \mid Z \in Z(F)\} = \bar{Z}(F)$.

LEMMA 3.1. *If* X *is* T_1, *then it is k-completely regular by* F *iff* Z (F) *is a basis of closed sets of* X. *If* X *is k-completely regular by* F *where k is locally compact, then* Z (F) *is a base of neighborhoods for* X.

Proof. Similar to classical proofs.

THEOREM 3.2. *Under condition* N, $\beta_F X$ *is homeomorphic to* W (Z, F).

Proof. θ is surjective, for suppose $\mathscr{F} \in W(Z, F)$. Let $\bar{\mathscr{F}}$ denote $\{\bar{Z} \mid Z \in \mathscr{F}\}$, then $\bar{\mathscr{F}}$ is a $\bar{Z}$-ultrafilter in $\beta_F X$. Since $\bar{\mathscr{F}}$ is a filter base in $\beta_F X$, we may choose $\mathscr{U}$, an ultrafilter such that $\mathscr{U} \supset \bar{\mathscr{F}}$, which must converge to $t_0 \in \beta_F X$ since $\beta_F X$ is compact, T_2. But then t_0 is an adherence point of $\bar{\mathscr{F}}$, hence of $\mathscr{F}$, thus $\mathscr{F} = \mathscr{F}(t_0) = \theta(t_0)$. θ is injective, for given $t, s \in \beta_F X$, $t \neq s$, there exists disjoint zeroset neighborhoods Z_t and Z_s in $\beta_F X$ such that $t \in V_t \subset Z_t$ and $s \in V_s \subset Z_s$, where V_t, V_s are open in $\beta_F X$ (by 3.1). Then $t \in V_t \cap \bar{X} \subset \overline{V_t \cap X}$, from which $t \in \overline{Z_t \cap X} = \bar{Z}_t$ showing that $Z_t \in \mathscr{F}(t)$. Similarly, $Z_s \in \mathscr{F}(s)$, and since Z_t and Z_s are disjoint, $\mathscr{F}(t) \neq \mathscr{F}(s)$. Next, it can be shown that the collection $\{\bar{Z} \mid Z \in Z(F)\}$ forms a base for the closed sets on $\beta_F X$, then since θ is a bijection, the identity $\theta(\bar{Z}) = \{\theta(t) \mid t \in \bar{Z}\} = \{\mathscr{F}(t) \mid t \in \bar{Z}\} = \{\mathscr{F}(t) \mid Z \in \mathscr{F}(t)\}$ displays a one-to-one correspondance between the closed sets in the topologies of $\beta_F X$ and W (Z, F). Thus θ and θ^{-1} are continuous.

Almost all that is needed for Condition N to be satisfied is that F be a normal algebra:

PROPOSITION 3.3. *Let* X *be k-completely regular by* F *where k is locally compact, and let* $Z_1, Z_2 \in Z(F)$. *If* F *is normal, then* $\bar{Z}_1 \cap \bar{Z}_2 = \overline{Z_1 \cap Z_2}$ *and* $Z_1 \cap Z_2 = \varnothing$ *implies that* $\bar{Z}_1 \cap \bar{Z}_2 = \varnothing$.

Proof. If $Z_1 \cap Z_2 = \varnothing$, there exists f such that $f(Z_1) = 0$ and $f(Z_2) = 1$. Thus $Z_1 \subset Z(f) \subset Z(f^\beta)$ and $Z_2 \subset Z(1-f) \subset Z((1-f)^\beta)$. Hence $\bar{Z}_1 \subset Z(f^\beta)$ and $\bar{Z}_2 \subset Z((1-f)^\beta)$. But then $f^\beta(\bar{Z}_1) = 0$ and $f^\beta(\bar{Z}_2) = 1$ from which $\bar{Z}_1 \cap \bar{Z}_2 = \varnothing$. Now, suppose $x \in \bar{Z}_1 \cap \bar{Z}_2$; to show that $x \in \overline{Z_1 \cap Z_2}$ it is sufficient to show that $Z \cap (Z_1 \cap Z_2) \neq \varnothing$ for each zeroset neighborhood Z of x (3.1). Let V be such that $x \in V \subset Z$. Since $x \in \bar{Z}_1 \cap \bar{Z}_2$ we have that $x \in V \cap \bar{Z}_1 \subset \subset Z \cap \bar{Z}_1$. But then $x \in \overline{V \cap Z_1} \subset \overline{Z \cap Z_1}$. Similarly $x \in \overline{Z \cap Z_2}$, and since $Z_1 \cap Z_2 = \varnothing$ implies $\bar{Z}_1 \cap \bar{Z}_2 = \varnothing$, we have $(Z \cap Z_1) \cap (Z \cap Z_2) \neq \varnothing$.

The final requirement is that the intersection of zerosets be a zeroset:

PROPOSITION 3.4. *If k is a nonalgebraically closed topological field, and* F *is an algebra of functions for* X *over k, then the intersection of two zerosets in* F *is a zeroset. Further, if* F *contains the inverse of each of its invertible functions, then* F *is a normal algebra.*

Proof. Since k is not algebraically closed, there exists an irreducible polynomial over k, $P = x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ with $n > 1$. Let $f, g \in F$ and define $h = f^n + a_{n-1} f^{n-1} g + \cdots + a_1 f g^{n-1} + a_0 g^n$. Then $Z(h) = Z(f) \cap Z(g)$ as is shown in [1] Them. 5. Now, if $Z(f)$ and $Z(g)$ are distinct zerosets it follows that $h(x) \neq 0$ for all $x \in X$, then h is invertible in F. Then define $k \in F$ as $k = f^n/h$. Since $k(x) = 0$ for all $x \in Z(f)$ and $k(x) = 1$ for all $x \in Z(g)$, F is normal.

PROPOSITION 3.5. *If k is locally compact and either nontrivially valued or nonalgebraically closed, and if* X *is* T_1 *and k-completely regular by a normal algebra* F, *then* $\beta_F X$ *is homeomorphic to* W (Z, F).

Proof. All that remains is to verify Condition N *b*), which is well known when k is R, C or H, and follows from 3.4 when k is nonalgebraically closed. If k is nonarchimedean and nontrivially valued, then k is discretely valued ([14] 1.4, Them. 2 Cor.), and if $r < 1$ is a generator of the value group of k then $x^2 - \rho$ is irreducible where $|\rho| = r$; thus k is nonalgebraically closed and 3.4 may be used.

When k is R, if F is inverse closed (F contains the inverse of all its invertible functions), then F is normal (3.4). Thus the β-like compactifications of Mrowka [12] are included among the $\beta_F X$ compactifications when F is normal. It would be of interest to see whether a function algebra must be inverse closed in order to be normal.

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