Atti Accademia Nazionale dei Lincei

Classe Scienze Fisiche Matematiche Naturali

Rendiconti

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On holomorphically subprojective complex manifolds

Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. **66** (1979), n.1, p. 12–18.

Accademia Nazionale dei Lincei

<http://www.bdim.eu/item?id=RLINA_1979_8_66_1_12_0>

Geometria differenziale. — *On holomorphically subprojective complex manifolds.* Nota di SHIZUKO SATO, presentata (*) dal Socio G. SANSONE.

RIASSUNTO. — Si discutono varie relazioni fra la classe delle varietà complesse olomorficamente sottoproiettive e quelle della varietà complesse H—S–proiettivamente piatte. Si considera in particolare il caso in cui le varietà in esame siano kähleriane.

In this paper, we shall discuss relations between holomorphically subprojective complex manifolds and HS-projectively flat complex manifolds and, in the last section, deal with Kählerian manifolds.

§ 1. Let M^{2m} be a complex manifold with a symmetric F-connection Γ^h_{ji}, i.e. a symmetric affine connection with respect to which the complex structure F is covariant constant. If there exists a complex coordinate system such that every holomorphically planar curve is given by m-2 homogeneous linear equations in this system and one other equation that need not be linear, then M is called a holomorphically subprojective complex manifold. The following theorem is known:

THEOREM A [6]. *M^{2m} $(m > 2)$ is a holomorphically subprojective complex manifold if and only if there exists a local real coordinate system (x^i) such that*

$$\Gamma^h_{ji} = \rho_{(j}\delta^h_{i)} + \tilde{\rho}_{(j}F^h_{i)} + f_{ji}\,x^h - f_{jr}\,F^r_i\,\tilde{x}^h \; (f_{[ji]} = f_{r[j}F^r_{i]} = 0)\,,$$

where ρ (resp. f) is a certain covariant vector field (resp. covariant tensor field) and we define $\tilde{\rho}_j = -F^r_j\,\rho_r$ and $\tilde{x}^h = F^h_r\,x^r$.

Now we assume that there is given a vector field v in M and consider a holomorphically subplanar curve $x^h = x^h(t)$ with respect to v, i.e.

$$\frac{d^2 x^h}{dt^2} + \Gamma^h_{ji}\frac{dx^j}{dt}\frac{dx^i}{dt} = \alpha(t)\frac{dx^h}{dt} + \beta(t)\,F^h_r\frac{dx^r}{dt} + \gamma(t)\,v^h + \varepsilon(t)\,\tilde{v}^h.$$

In [6], it is known that two symmetric F-connections Γ^h_{ji} and $\overline{\Gamma}^h_{ji}$ have all holomorphically subplanar curves with respect to the same vector field v in common if and only if

$$\overline{\Gamma}^h_{ji} = \Gamma^h_{ji} + \rho_{(j}\delta^h_{i)} + \tilde{\rho}_{(j}F^h_{i)} + f_{ji}\,v^h - f_{jr}\,F^r_i\,\tilde{v}^h \; (f_{[ji]} = f_{r\,[j}F^r_{i]} = 0)\,.$$

This correspondence $\overline{\Gamma}^h_{ji} \rightarrow \Gamma^h_{ji}$ is called a holomorphically subprojective transformation. Specially, a complex manifold with a symmetric F-con-

(*) Nella seduta del 16 dicembre 1978.

nection which is obtained from a complex linear space C^m by a holomorphically subprojective transformation is called an HS-projectively flat complex manifold.

Finally, let $\underset{v}{\mathrm{L}}$ and ∇_j be the operators of Lie differentiation with respect to v and the covariant differentiation with respect to a given connection. Then a vector field v is called an HSP-transformation if it satisfies

$$(1.1)\qquad \underset{v}{\mathrm{L}}\,\Gamma^h_{ji} = \psi_{(j}\delta^h_{i)} + \tilde{\psi}_{(j}F^h_{i)} + \psi_{ji}v^h - \psi_{jr}F^r_i\tilde{v}^h\ (\psi_{[ji]} = \psi_{r[j}F^r_{i]} = 0)$$

and if it satisfies

$$(1.2)\qquad \nabla_j v^h = a\delta^h_j + bF^h_j + \alpha_j v^h + \tilde{\alpha}_j\tilde{v}^h\,,$$

then it is called contravariant analytic almost K-torse-forming.

§ 2. Let M^{2m} $(m > 2)$ be an HS-projectively flat complex manifold. Then the symmetric F-connection Γ^h_{ji} takes the form, for a suitable coordinate system (x^i),

$$(2.1)\qquad \Gamma^h_{ji} = 2\,\varphi_{(j}\delta^h_{i)} + 2\,\tilde{\varphi}_{(j}F^h_{i)} + \varphi_{ji}v^h - \varphi_{jr}F^r_i\tilde{v}^h\ (\varphi_{[ji]} = \varphi_{r\,[j}F^r_{i]} = 0)\,,$$

where φ_j (resp. φ_{ji}) is a certain covariant vector field (resp. covariant tensor field).

In this section, we assume that the vector field v^h is an analytic almost K-torse-forming one satisfying (1.2). The curvature tensor $R_{jik}{}^h$ of Γ^h_{ji} can be written as below:

$$(2.2)\qquad \begin{aligned} R_{jik}{}^h = &-2\,(u_{k\,[j}\delta^h_{i]} - F^r_k u_{r\,[j}F^h_{i]} + U_{kji}v^h - F^r_k U_{rji}\tilde{v}^h \\ &-\nabla_{[j}\varphi_{i]}\,\delta^h_k - \nabla_{[j}\tilde{\varphi}_{i]}\,F^h_k)\,, \end{aligned}$$

where we put

$$(2.3)\qquad u_{ji} = -\nabla_i\varphi_j - \varphi_j\varphi_i + \tilde{\varphi}_j\tilde{\varphi}_i + (a - v^r\varphi_r)\varphi_{ji} + (b - v^r\tilde{\varphi}_r)\varphi_{js}F^s_i\,,$$

$$(2.4)\qquad U_{kji} = -\nabla_{[j}\varphi_{i]k} - \alpha_{[j}\varphi_{i]k} - \tilde{\alpha}_{[j}\varphi_{i]r}F^r_k + \varphi_{r\,[j}\varphi_{i]k}v^r - \varphi_{r\,[j}\varphi_{i]s}F^s_k\tilde{v}^r\,.$$

Now we consider the differential equation

$$(2.5)\qquad \nabla_j z_i = -2\,z_{(j}\varphi_{i)} + 2\,\tilde{z}_{(j}\tilde{\varphi}_{i)} - z_r v^r\varphi_{ji} + z_r\tilde{v}^r\varphi_{js}F^s_i\,.$$

By virtue of (1.2) and (2.2) $\sim$ (2.5), we obtain

$$\nabla_k\nabla_j z_i - \nabla_j\nabla_k z_i = -R_{kji}{}^r z_r\,,$$

which means that the integrability condition of (2.5) is satisfied identically and then there exist $2m$ linearly independent solutions $z^{(a)}_i$. Since $\nabla_{[j}z_{i]} = 0$, $z^{(a)}_i$ are gradient covariant vectors and if we put $z^{(a)}_i = \dfrac{\partial\bar{x}^a}{\partial x^i}$, $\bar{x}^a = \bar{x}^a(x^i)$ are $2m$ linearly independent functions defining a transformation of coordinates.

Thus, by the transformation law under $\bar{x}^a = \bar{x}^a(x^i)$, it is easily seen that

$$\bar{\Gamma}^a_{bc} = -\frac{\partial x^j}{\partial \bar{x}^b}\frac{\partial x^i}{\partial \bar{x}^c}\nabla_j z_i^{(a)} = 2\,\bar{\varphi}_{(b}\delta^a_{c)} + 2\,\bar{\tilde{\varphi}}_{(b}\bar{F}^a_{c)} + \bar{\varphi}_{bc}\,\bar{v}^a - \bar{\varphi}_{bd}\,\bar{F}^d_c\,\bar{\tilde{v}}^a,$$

where $\bar{\varphi}_b$, etc. are components in coordinates $(\bar{x}^a)$. We shall prove

THEOREM 1. *Let* M^{2m} $(m > 2)$ *be an* HS-*projectively flat complex manifold with a symmetric* F-*connection given by* (2.1). *Then* M *is a holomorphically subprojective complex manifold under the following conditions*:

(1) v^h *in* (2.1) *is an analytic almost* K-*torse-forming vector field satisfying* (1.2),

(2) v^h *and* $\tilde{v}^h$ *are* HSP-*transformations*,

where $(a - v^r\varphi_r)^2 + (b - v^r\tilde{\varphi}_r)^2 \neq 0$.

Proof. We put $f = a - v^r\varphi_r$ and $g = b - v^r\tilde{\varphi}_r$. Since v^h is an HSP-transformation, in [4] it is known that

$$(2.6)\qquad a_k - a\alpha_k + b\tilde{\alpha}_k + u_{rk}v^r = 0 \quad , \quad b_k - a\tilde{\alpha}_k - b\alpha_k - u_{rk}\tilde{v}^r = 0\ ^{(1)},$$

which imply that $\tilde{f}_j = g_j$ holds good by virtue of (1.2) and (2.3). Hence, from (1.1) and conditions, $\xi^h = (1/f^2 + g^2)(fv^h - g\tilde{v}^h)$ is an HSP-transformation and an analytic almost K-torse-forming vector field satisfying

$$\nabla_j\xi^h = a'\,\delta^h_j + b'\,F^h_j + (\alpha_j - \gamma_j)\,\xi^h + (\tilde{\alpha}_j - \tilde{\gamma}_j)\,\tilde{\xi}^h,\ (\gamma_j \overset{\text{put}}{=} (1/f^2 + g^2)$$
$$(ff_j + gg_j)),\ \xi^r\varphi_r - a' = -1 \quad , \quad \xi^r\tilde{\varphi}_r - b' = 0,$$

from which we may assume that

$$(2.7)\qquad a = v^r\varphi_r + 1 \quad \text{and} \quad b = v^r\tilde{\varphi}_r$$

and then taking account of (2.3) and (2.6), we get

$$(2.8)\qquad \alpha_k = \varphi_k + \varphi_{kr}v^r.$$

Now we prove that the solutions $z_j^{(a)} = \dfrac{\partial\bar{x}^a}{\partial x^j}$ of (2.5) satisfies $z_r^{(a)}v^r = \bar{x}^a$. Making use of (1.2) and (2.5), we have

$$\nabla_k(z_r v^r) = (a - v^r\varphi_r)\,z_k - (b - v^r\tilde{\varphi}_r)\,z_k + z_r v^r(\alpha_k - \varphi_k - \varphi_{ks}v^s)$$
$$+ z_r\tilde{v}^r(\tilde{\alpha}_k - \tilde{\varphi}_k + F^t_k\varphi_{ts}v^s)$$

and substituting (2.7) and (2.8), we obtain that $\nabla_k(z_r v^r) = z_k$, which implies that $z_r^{(a)}v^r = \bar{x}^a$ and $z_r^{(a)}\tilde{v}^r = \bar{\tilde{x}}^a$, i.e. $\bar{v}^a = \bar{x}^a$ and $\bar{\tilde{v}}^a = \bar{\tilde{x}}^a$.

Hence we have $\bar{\Gamma}^a_{bc} = 2\,\bar{\varphi}_{(b}\delta^a_{c)} + 2\,\bar{\tilde{\varphi}}_{(b}\bar{F}^a_{c)} + \bar{\varphi}_{bc}\,\bar{x}^a - \bar{\varphi}_{bd}\,\bar{F}^d_c\,\bar{\tilde{x}}^a$ and proved Theorem 1 by virtue of Theorem A. q.e.d.

(1) For a function f, we put $\nabla_j f = f_j$.

§ 3. In this section, we consider an HS-projectively flat Kählerian manifold M^{2m} ($m > 2$) and replace the symmetric F-connection Γ^h_{ji} in the preceding section with the Christoffel symbol $\left\{ {h \atop ji} \right\}$. Also we assume that v^h is an analytic K-torse-forming vector field satisfying (1.2). Then $\nabla_{[j}\varphi_{i]} = 0$ is valid in [8] and v^h and $\tilde{v}^h$ are analytic HSP-transformations, see [5, 7].

Now we consider a transformation of φ_j in (2.1). If $\left\{ {h \atop ji} \right\}$ may be transformable to the form, by a suitable transformation $\bar{x}^a = \bar{x}^a(x^i)$ of coordinates,

$$(3.1)\qquad \overline{\left\{ {d \atop bc} \right\}} = 2\,\overline{\psi}_{(b}\delta^a_{c)} + 2\,\bar{\tilde{\psi}}_{(b}\overline{F}^a_{c)} + \overline{\psi}_{bc}\,\bar{v}^a - \overline{\psi}_{bd}\,\overline{F}^d_c\,\bar{\tilde{v}}^a \qquad (\overline{\psi}_{[bc]} = \overline{\psi}_d\,{}_{[b}\overline{F}^d_{c]} = 0),$$

then we indicate quantities corresponding to $R_{jik}{}^h$, u_{ji} and U_{kji} in § 2 by $\bar{R}_{jik}{}^h$, $\bar{h}_{ji}$ and $\overline{V}_{kji}$. We have the following

LEMMA 3.1. *If* $\left\{ {h \atop ji} \right\}$ *given by* (2.1) *is transformed to the form of* (3.1) *by a suitable transformation of coordinates, then* $u_{ji} = h_{ji}$ *and* $U_{kji} = V_{kji}$.

Proof. We put $\phi_j = \varphi_j - \psi_j$, $w_{ji} = u_{ji} - v_{ji}$ and $W_{kji} = U_{kji} - V_{kji}$ and then we have

$$(3.2)\qquad w_{[ji]} = \nabla_{[j}\phi_{i]} = 0 \quad , \quad w_{r\,[j}F^r_{i]} = \nabla_{[j}\tilde{\phi}_{i]}\,,$$

$$(3.3)\qquad w_{rj}\,v^r + F^s_j\,w_{sr}\,v^r = 2\,\nabla_{[r}\phi_{j]}\,\tilde{v}^r = -2\,\nabla_{[r}\tilde{\phi}_{s]}\,v^r\,F^s_j\,,$$

$$(3.4)\qquad -w_{k\,[j}\delta^h_{i]} + F^r_k\,w_{r\,[j}F^h_{i]} - W_{kji}\,v^h + F^r_k\,W_{rji}\,\tilde{v}^h + \nabla_{[j}\tilde{\phi}_{i]}\,F^h_k = 0\,.$$

At first we prepare necessary formulas by making use of (3.3)~(3.4).

$$(3.5)\qquad (2m-1)\,w_{ki} + F^r_k\,F^s_i\,w_{rs} + 2\,(W_{kir}\,v^r - F^s_k\,W_{sir}\,\tilde{v}^r + \nabla_{[s}\tilde{\phi}_{i]}\,F^s_k) = 0\,,$$

$$(3.6)\qquad (m+1)\,\nabla_{[j}\phi_{i]} = W_{rji}\,v^r = 0 \quad , \quad (m+1)\,\nabla_{[j}\tilde{\phi}_{i]} = -W_{rji}\,\tilde{v}^r\,,$$

$$(3.7)\qquad \begin{aligned} &2\,(W_{kir}\,v^r + F^s_k\,W_{sir}\,\tilde{v}^r)\,v^h + 2\,(W_{kir}\,\tilde{v}^r - F^s_k\,W_{sir}\,v^r)\,\tilde{v}^h \\ &\quad -(w_{kr}\,v^r + F^s_k\,w_{sr}\,\tilde{v}^r)\,\delta^h_i + (F^s_k\,w_{sr}\,v^r - w_{kr}\,\tilde{v}^r)\,F^h_i \\ &\quad -2\,(\nabla_{[r}\tilde{\phi}_{i]}\,\tilde{v}^r\,\delta^h_k - \nabla_{[r}\tilde{\phi}_{i]}\,v^r\,F^h_k) = 0\,, \end{aligned}$$

$$(3.8)\qquad 2\,(m-2)\,\nabla_{[r}\tilde{\phi}_{k]}\,\tilde{v}^r + (w_{kr}\,v^r + F^s_k\,w_{sr}\,\tilde{v}^r - 2\,F^s_k\,W_{srt}\,v^r\,\tilde{v}^t) = 0\,,$$

where we use $W_{k(ij)} = 0$.

Next we choose any 1-form η_i orthogonal to v and put $\eta = \eta_r\,\tilde{v}^r$. If we transvect (3.7) with η_h, then we have

$$(3.9)\qquad \begin{aligned} &2\,\eta\,(W_{kir}\,\tilde{v}^r - F^s_k\,W_{sir}\,v^r) - (w_{kr}\,v^r + F^s_k\,w_{sr}\,\tilde{v}^r)\,\eta_i \\ &\quad -(F^s_k\,w_{sr}\,v^r - w_{kr}\,\tilde{v}^r)\,\tilde{\eta}_i - 2\,(\nabla_{[r}\tilde{\phi}_{i]}\,\tilde{v}^r\,\eta_k + \nabla_{[r}\tilde{\phi}_{i]}\,v^r\,\tilde{\eta}_k) = 0\,, \end{aligned}$$

from which it follows that $F_k^s w_{sr} v^r - w_{kr} \tilde{v}^r + 2 W_{krt} v^r \tilde{v}^t = 0$ by virtue of (3.2), (3.3), (3.6) and $W_{k(ji)} = 0$ and moreover from (3.8) we get $w_{kr} v^r = w_{kr} \tilde{v}^r = \nabla_{[r}\tilde{\phi}_{k]} \tilde{v}^r = \nabla_{[r}\tilde{\phi}_{k]} v^r = 0$. Substituting the last equation into (3.9), it is clear that

$$(3.10) \qquad F_k^r w_{ri} + 2 F_k^s W_{sir} v^r = 0 \quad , \quad W_{kir} \tilde{v}^r - F_k^s W_{sir} v^r = 0 ,$$

which imply that $w_{r[i} F_{k]}^r + 2 W_{[ki]r} \tilde{v}^r = 0$ and comparing with (3.6), $\nabla_{[i}\tilde{\phi}_{k]} = 0$ holds good.

Hence we obtain the desired equations $w_{ki} = 0$ and $W_{kji} = 0$ by virtue of (3.2), (3.4), (3.5) and (3.10). q.e.d.

In the case of Lemma 3.1, we have $\nabla_{[j}\varphi_{i]} = \nabla_{[j}\psi_{i]} = 0$ and $\nabla_{[j}\tilde{\varphi}_{i]} = \nabla_{[j}\tilde{\psi}_{i]}$. If we put $z_i^{(a)} = \dfrac{\partial \bar{x}^a}{\partial x^i}$ for the transformation $\bar{x}^a = \bar{x}^a(x^i)$ of coordinates, then we have

$$\begin{aligned} \overline{\left\{ {a \atop bc} \right\}} &= 2 \bar{\psi}_{(b}\delta_{c)}^a + 2 \bar{\tilde{\psi}}_{(b}\bar{F}_{c)}^a + \bar{\psi}_{bc} \bar{v}^a - \bar{\psi}_{bd} \bar{F}_c^d \bar{\tilde{v}}^a \\ &= \frac{\partial x^j}{\partial \bar{x}^b} \frac{\partial x^i}{\partial \bar{x}^c} \left(\left\{ {h \atop ji} \right\} \frac{\partial \bar{x}^a}{\partial x^h} - \frac{\partial^2 \bar{x}^a}{\partial x^j \partial x^i} \right) , \end{aligned}$$

from which it follows that

$$\nabla_j z_i^{(a)} + 2 z_{(j}^{(a)} \psi_{i)} - 2 \tilde{z}_{(j}^{(a)} \tilde{\psi}_{i)} + v^r z_r^{(a)} \psi_{ji} - \tilde{v}^r z_r^{(a)} \psi_{js} F_i^s = 0 .$$

Thus it is easily seen that $2m$ covariant vectors $z_i^{(a)}$ must satisfy

$$(3.11) \qquad \nabla_j z_i = - 2 z_{(j}\psi_{i)} + 2 \tilde{z}_{(j}\tilde{\psi}_{i)} - v^r z_r \psi_{ji} + \tilde{v}^r z_r \psi_{js} F_i^s$$
$$(\psi_{[ji]} = \psi_{r[j} F_{i]}^r = 0).$$

In the following, we consider a converse of Lemma 3.1.

LEMMA 3.2. *If we choose any closed* 1*-form* ψ_j *such that*

$$(3.12) \qquad (a - v^r \psi_r)^2 + (b - v^r \tilde{\psi}_r)^2 \neq 0 \quad , \quad \nabla_{[j}\tilde{\psi}_{i]} = \nabla_{[j}\tilde{\varphi}_{i]} ,$$

then $\left\{ {h \atop ji} \right\}$ *given by* (2.1) *can be transformed to the form of* (3.1) *by a suitable transformation of coordinates.*

Proof. We define ψ_{ji} by

$$(3.13) \qquad h_{ji} = u_{ji}$$

and then $\psi_{[ji]} = \psi_{r[j} F_{i]}^r = 0$ hold good by virtue of (3.12). Hence the integrability condition of (3.11) is

$$(3.14) \qquad - R_{jik}^{\ \ \ r} z_r = 2 (h_{k[j}\delta_{i]}^s - F_k^r h_{r[j} F_{i]}^s + V_{kji} v^s - F_k^r V_{rji} \tilde{v}^s - \nabla_{[j}\tilde{\psi}_{i]} F_k^s) z_s .$$

Here, with straightforward calculation, we obtain

$$(3.15)\qquad \nabla_j h_{ki} - \nabla_i h_{kj} = R_{jik}{}^r \psi_r + 2\,(h_{k[j}\psi_{i]} + F_k^r h_{r[j}\tilde{\psi}_{i]} + \tilde{\psi}_k \nabla_{[j}\tilde{\psi}_{i]})$$
$$-\,2\,((a - v^r\psi_r)\,V_{kji} + (b - v^r\tilde{\psi}_r)\,F_k^s V_{sji} + \psi_{k[j}\beta_{i]} - \psi_{kr}\,F_{[j}^r\gamma_{i]})\,,$$

where we put

$$\beta_i = a_i - a\alpha_i + b\tilde{\alpha}_i + h_{ri}v^r \qquad \text{and} \qquad \gamma_i = b_i - a\tilde{\alpha}_i - b\alpha_i - h_{ri}\tilde{v}^r\,.$$

Since v is an HSP-transformation, we have

$$(3.16)\qquad \nabla_j u_{ki} - \nabla_i u_{kj} = -\,2\,a\,U_{kji} - 2\,b\,F_k^r U_{rji}\,.$$

Substituting (2.2), (2.6), (3.12) and (3.13) into (3.15), we obtain

$$\nabla_j u_{ki} - \nabla_i u_{kj} = -\,2\,(v^r\psi_r U_{kji} + v^r\tilde{\psi}_r F_k^s U_{sji}$$
$$+\,(a - v^r\psi_r)\,V_{kji} + (b - v^r\tilde{\psi}_r)\,F_k^s V_{sji})\,,$$

from which and (3.16) it follows that $U_{kji} = V_{kji}$ is valid by virtue of (3.12).

Hence the equation of (3.14) is satisfied identically and if we denote $2m$ linearly independent solutions by $z_i^{(a)} = \dfrac{\partial\bar{x}^a}{\partial x^i}$, then $\bar{x}^a = \bar{x}^a(x^i)$ is the desired transformation of coordinates. q.e.d.

As a consequence of Theorem 1 and Lemma 3.2, we have

COROLLARY. *An* HS-*projectively flat Kählerian manifold* M^{2m} $(m > 2)$ *is a holomorphically subprojective Kählerian manifold if* v^h *appearing on the Christoffel symbol is an analytic* K-*torse-forming vector field.*

LEMMA 3.3. *If we choose* a 1-*form* ψ_j *such that*

$$(3.17)\qquad (a - v^r\psi_r)^2 + (b - v^r\tilde{\psi}_r)^2 = 0\,,$$

then $\left\{ {h \atop ji} \right\}$ *given by* (2.1) *can be transformed to the form of* (3.1) *by a suitable transformation of coordinates.*

Proof. In this case, $h_{ji} = -\nabla_i\psi_j - \psi_j\psi_i + \tilde{\psi}_j\tilde{\psi}_i$ and at first we consider the following

$$(3.18)\qquad -\nabla_i\psi_j - \psi_j\psi_i + \tilde{\psi}_j\tilde{\psi}_i = u_{ji}\,,$$

whose the integrability condition is, by virtue of (3.16),

$$-\,R_{kij}{}^r\psi_r = 2\,(u_{j[k}\psi_{i]} + F_j^r u_{r[k}\tilde{\psi}_{i]} + \tilde{\psi}_j\nabla_{[k}\tilde{\psi}_{i]} - aU_{jik} - bF_j^r U_{rik})\,,$$
$$\nabla_k(a - v^r\psi_r) = 0 \quad , \quad \nabla_k(b - v^r\tilde{\psi}_r) = 0\,.$$

From (2.2), (2.6), (3.17) and (3.18), the above equations are satisfied identically.

Next we assume that ψ_{ji} satisfies $V_{kji} = U_{kji}$ (e.g. $\psi_{ji} \overset{\text{put}}{=} \varphi_{ji}$).

Hence the integrability condition of (3.11), i.e. (3.14), is satisfied identically and the rest of proof is the same way as Lemma 3.2. q.e.d.

According to Lemma 3.1$\sim$3.3, we have

THEOREM 2. *In an* HS-*projectively flat Kählerian manifold* M^{2m} $(m > 2)$, *in order that the Christoffel symbol given by* (2.1) *may be transformed to the form of* (3.1), *it is necessary and sufficient that* $u_{ji} = h_{ji}$ *and* $U_{kji} = V_{kji}$ *hold good, where the given vector field* v^h *is analytic* K-*torse-forming*.

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