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Linear response of the gate system for protection of the Venice Lagoon. Note I: Transverse free modes

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Meccanica dei fluidi. — *Linear response of the gate system for protection of the Venice Lagoon. Note I: Transverse free modes.* Nota di Paolo Blondeaux, Giovanni Seminara e Giovanna Vittori, presentata (*) dal Socio E. Marchi.

Abstract. — The free oscillations of the gate system proposed [1, 2] to defend the Venice Lagoon from the phenomenon of high water are analyzed. Free transverse modes of oscillations exist which may be either subharmonic or synchronous with respect to typical waves in the Adriatic sea. This result points out the need to examine whether such modes may be excited as a result of a Mathieu type resonance occurring when the gate system is forced by incident waves. The latter investigation is performed in part 2 of the present paper.

Key words: Gates; Waves; Free oscillations.

Riassunto. — *Risposta lineare della schiera di ventole per la protezione della Laguna Veneta. Nota I: Modi trasversali liberi.* Si analizzano le oscillazioni libere della schiera di ventole oscillanti proposte [1, 2] per la difesa della Laguna di Venezia dalle acque alte. Si evidenzia l'esistenza di possibili modi liberi trasversali sia subarmonici che sincroni rispetto alle tipiche onde incidenti del mare Adriatico. Il risultato pone l'esigenza di verificare se tali modi possono essere eccitati come conseguenza di una risonanza alla Mathieu quando la schiera di ventole è forzata da onde incidenti normalmente alle ventole. Questo studio è effettuato nella seconda parte del presente lavoro.

1. Introduction

The protection system designed [1, 2] to defend Venice from the increasingly recurrent phenomenon of high water essentially consists of gate systems able to stop tidal currents from flowing through each of the three inlets of Venice Lagoon (Bocca di Lido, Bocca di Malamocco and Bocca di Chioggia). Gates which are normally submerged and lie horizontally on the bottom are lifted up during flood events and are allowed to oscillate around their equilibrium positions under the action of incoming waves. If the responses of the gates are spatially in phase, the presence of the gates makes the Lagoon practically unaffected by tidal oscillations in the Adriatic sea. However experiments performed at the Delft Hydraulics Laboratory [3] have shown that a variety of responses of the gate system is possible even when the incoming wave is purely monochromatic with propagation velocity perfectly orthogonal to the gates. Some of these responses were characterized by a frequency different from that of the incoming waves or by oscillations of the gates which were not in phase.

In the present contribution we investigate the mechanism controlling the various types of responses which may arise as the frequency and amplitude of the incoming wave vary for given values of the flow depth and channel width.

(*) Nella seduta del 18 giugno 1993.

Since our aim is to disclose the basic mechanism, we will introduce some simplifications into the problem such to make the latter more easily amenable to analytical treatment while not affecting the qualitative behaviour of the response. The main simplifying assumptions will be:

i) waves are assumed to be inviscid and to propagate on shallow water;

ii) the gates are modeled as plane vertical walls allowed to slide along the bottom in the direction parallel to the channel axis, while the recoil effect associated with Archimede's force acting on the actual gates is modelled by introducing springs with constant stiffness k^*;

iii) the number of gates is assumed to be sufficiently large for any transverse mode of oscillations to be allowed.

Assumption *i*) is not wholly justified as waves in the Adriatic sea have typical wavelengths ranging about 100 m while the mean flow depth in the inlet channels varies between 10 and 15 m. However 3-*D* effects are fairly small and quite unlikely may change the qualitative pattern of the response, while complicating the analysis considerably.

Assumption *ii*) makes the motion of the gates compatible with the 2-*D* representation of water waves. In the actual configuration the recoil force is nonlinearly related to the angular displacement of the gate from the equilibrium position. Again this appears to be a minor effect which could be modelled by allowing the stiffness k^* to depend on the horizontal displacement of the vertical gates.

The procedure employed in the rest of the paper is as follows. In the next section we present the mathematical formulation of the problem. In section 3 we derive the basic solution corresponding to a purely monochromatic incoming wave generating an in-phase response of the gate system. Section 4 is devoted to a linear stability analysis of the equilibrium configuration against free transverse perturbations both of the flow field and of gate response. Some results and discussion follow in section 5.

Part 2 of the present paper tackles the relevant problem of determining the mechanism whereby the forcing effect of the incoming wave assumed to propagate in the direction orthogonal to the gate system may lead to excitation of the free modes analyzed in part 1.

2. Mathematical formulation of the problem

Let us consider a straight infinitely long channel of width B^* (a star denotes dimensional quantities) and introduce longitudinal and transverse coordinates, x^* and y^* respectively, as depicted in fig. 1.

At $x^* = x^*_{GO}$ an infinite sequence of vertical gates characterized by infinitesimally small width is in equilibrium in the undisturbed configuration with x^*_{GO} vanishing if the water surface elevations on the two sides of the gate system are equal. As discussed in the introduction, each gate is allowed to slide horizontally along the bottom and is subject to a recoil force due to the action of a spring with constant stiffness k^*.

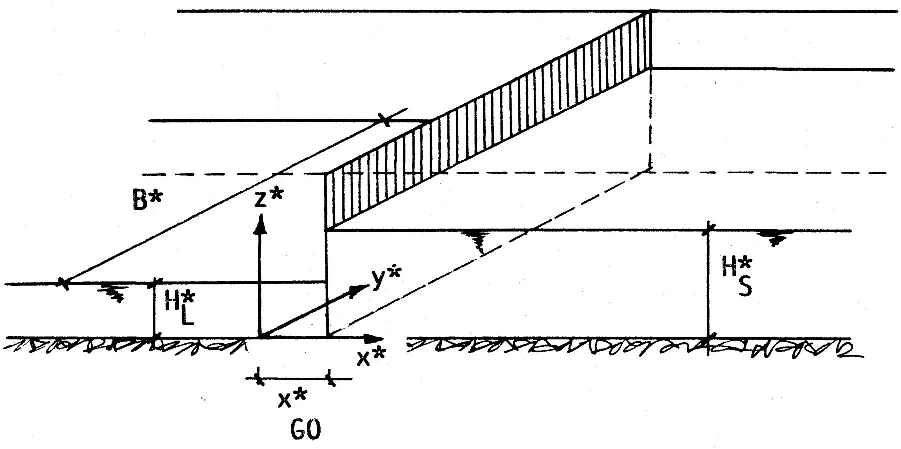

Fig. 1. – Sketch of the model.

We investigate the flow pattern and the related motion of the gates subject to the action of a purely monochromatic wave with angular frequency ω^* and amplitude a^* propagating from $x^* \to \infty$ in the direction orthogonal to the gates.

Let us introduce dimensionless quantities as follows

$$(x, y) = (x^*, y^*)/(\sqrt{gH_S^*}/\omega^*), \qquad t = \omega^* t^* \tag{1a}$$

$$(u, v) = (u^*, v^*)/(\sqrt{gH_S^*}(a^*/H_S^*)), \qquad \eta = \eta^*/a^* \tag{1b}$$

where H_S^* denotes the constant average flow depth in the «sea» region ($x > x_G$), η^* is the local water surface elevation relative to the undisturbed value, u^*, v^* are depth averaged velocity components in the longitudinal and transverse direction respectively. The suffix L will denote quantities related to the «lagoon» region ($x < x_G$) while the suffix S will refer to the «sea» region.

Neglecting viscous dissipation, the flow field is then readily shown to be governed by the following differential problem

Lagoon

$$(h_L + a\eta_L)_{,t} + a[(h_L + a\eta_L)u_L]_{,x} + a[(h_L + a\eta_L)v_L]_{,y} = 0\,, \tag{2}$$

$$u_{L,t} + au_L u_{L,x} + av_L u_{L,y} = -\eta_{L,x}\,, \tag{3}$$

$$v_{L,t} + au_L v_{L,x} + av_L v_{L,y} = -\eta_{L,y}\,, \tag{4}$$

$$v_L = 0 \qquad (y = 0,\ \beta;\ -\infty < x \leqslant x_G)\,, \tag{5}$$

$$u_L + v_L x_{G,y} = x_{G,t} a^{-1} (x = x_G;\ 0 \leqslant y \leqslant \beta)\,, \tag{6}$$

where a, β and h_L are the following dimensionless parameters

$$a = a^*/H_S^*\,, \qquad \beta = B^*/(\sqrt{gH_S^*}/\omega^*)\,, \qquad h_L = H_L^*/H_S^* \tag{7}$$

and x_G denotes the dimensionless value of the longitudinal coordinate at which the gate is instantaneously located. We point out that x_G is in general a function of y and t.

Sea

In the sea region the governing system is identical with (2)-(6) except for the suffix S replacing L and 1 replacing h_L.

Gate motion

Finally the flow fields in the L-and S-regions must be coupled through the equation of motion of the gate system which can be written in the following dimensionless form

$$mx_{G,tt} = -kx_G + (1/2)\{(h_L + a\eta_L)^2 - (1 + a\eta_S)^2\}_{x=x_G} \tag{8}$$

where m and k are the following dimensionless parameters

$$m = m^* \omega^*/(\rho H_S^* \sqrt{gH_S^*}), \qquad k = k^*/(\omega^* \rho H_S^* \sqrt{gH_S^*}) \tag{9}$$

m^* being the gate mass per unit width.

3. Basic synchronous response of the gate system to a monochromatic excitation

Let us now expand the solution of the problem formulated in the previous section in the form

$$(u_L, v_L, \eta_L) = (u_{L0}, 0, \eta_{L0}) + O(a), \tag{10a}$$

$$(u_S, v_S, \eta_S) = (u_{S0}, 0, \eta_{S0}) + O(a), \tag{10b}$$

$$x_G = x_{G0} + ax_{G1} + O(a^2), \tag{10c}$$

where all the functions on the right hand side of (10*a, b, c*) are assumed to be independent of y. This corresponds to assuming the gates to undergo oscillations which are in phase in the transverse direction.

Substituting from (10) into the governing equations and equating likewise powers of a, at leading order we find:

$$\eta_{L0,tt} - h_L \eta_{L0,xx} = 0, \tag{11a}$$

$$u_{L0,t} = -\eta_{L0,x}, \tag{11b}$$

$$\eta_{S0,tt} - \eta_{S0,xx} = 0, \tag{11c}$$

$$u_{S0,t} = -\eta_{S0,x}, \tag{11d}$$

$$u_{L0}|_{x=x_{GO}} = x_{G1,t}, \tag{11e}$$

$$u_{S0}|_{x=x_{GO}} = x_{G1,t}, \tag{11f}$$

$$mx_{G1,tt} + kx_{G1} = +h_L \eta_{L0}|_{x_{G0}} - \eta_{S0}|_{x_{GO}}, \tag{11g}$$

where

$$x_{G0} = (h_L^2 - 1)/2k. \tag{12}$$

At this stage it is convenient to set

$$X = x - x_{G0} \tag{13}$$

so that the solution of ($11a$) reads

$$\eta_{L0} = A_L \exp i(\lambda X + t) + c.c. \,, \tag{14a}$$

$$u_{L0} = -\lambda \eta_{L0} \,, \tag{14b}$$

$$\eta_{S0} = \exp i(X + t) + B_S \exp i(X - t) + c.c. \,, \tag{14c}$$

$$u_{S0} = -\exp i(X + t) + B_S \exp i(X - t) + c.c. \,, \tag{14d}$$

with

$$\lambda = \sqrt{1/h_L} \tag{15}$$

and having set equal to 1 the dimensionless amplitude of the incident wave.

The values of A_L and B_S are readily determined by setting

$$x_{G1} = \mathcal{A} \exp(it) + c.c. \tag{16}$$

Substituting (16) into ($11e, f, g$) we find

$$\mathcal{A} = 2/(m - k - i(1 + h_L^{3/2})) \,, \tag{17}$$

$$-\lambda A_L = i\mathcal{A} \,, \tag{18}$$

$$-1 + \overline{B}_S = i\mathcal{A} \,. \tag{19}$$

The relationships (14)-(19) completely determine the basic solution. In the above formulas *c.c.* as well as an overbar denote the complex conjugate of a complex number.

4. Free transverse modes

We now examine the stability of the basic solution derived in section 3 against linear perturbations, both of the flow field and of gate alignement. Such perturbations are assumed to be periodic in the transverse direction. We then set

$$(u_L, v_L, \eta_L) = (u_{L0}, O, \eta_{L0}) + \varepsilon(f_L, g_L, e_L) + O(a, \varepsilon^2) \,, \tag{20a}$$

$$(u_S, v_S, \eta_S) = (u_{S0}, O, \eta_{S0}) + \varepsilon(f_S, g_S, e_S) + O(a, \varepsilon^2) \,, \tag{20b}$$

$$x_G - x_{G0} = a[x_{G1} + \varepsilon\zeta + O(a, \varepsilon^2)] \,, \tag{20c}$$

with ε infinitesimal amplitude.

Substituting from ($20a,b,c$), into the governing differential systems and performing linearization in ε we find the following problems.

Lagoon

$$e_{L,t} + h_L f_{L,X} + h_L g_{L,y} = -a[(\eta_{L0} f_L)_{,X} + (e_L u_{L0})_{,X} + (\eta_{L0} g_L)_{,y}] \,, \tag{21a}$$

$$f_{L,t} + e_{L,X} = -a[u_{L0} f_{L,X} + f_L u_{L0,X}] \,, \tag{21b}$$

$$g_{L,t} + e_{L,y} = -a[u_{L0} g_{L,X}] \,, \tag{21c}$$

$$f_L = \zeta_{,t}(X = a x_{G1} + O(a^2)) \,. \tag{21d}$$

Sea

A problem identical to (20) is found but for h_L replaced by 1 and the suffix S replacing L.

Gate motion

(22) $$m\zeta_{,tt} + k\zeta = [h_L e_L - e_S + a(\eta_{L0} e_L - \eta_{S0} e_S + h_L e_{L,X} x_{G1} - e_{S,X} x_{G1})]_{X=0}.$$

The structure of the above problem suggests the opportunity to expand the solution for the vector $V = (e_L, f_L, g_L, e_S, f_S, g_S, \zeta)$ in the form

(23) $$V = V_o + aV_1 + O(a^2).$$

Substituting from (23) into (20-22) at order a^o we find a differential problem governing the free response of the gate system to transverse oscillations.

The $O(\varepsilon a^o)$ problem is readily reduced to the following form

(24a) $$e_{L0,tt} - h_L(e_{L0,XX} + e_{L0,yy}) = 0,$$

(24b) $$f_{L0,t} = -e_{L0,X},$$

(24c) $$g_{L0,t} = -e_{L0,y},$$

(24d) $$e_{S0,tt} - e_{S0,XX} - e_{S0,yy} = 0,$$

(24e) $$f_{S0,t} = -e_{S0,X},$$

(24f) $$g_{S0,t} = -e_{S0,y},$$

(24g) $$m\zeta_{0,tt} + k\zeta_0 = h_L e_{L0} - e_{S0}.$$

Recalling the boundary conditions we set

(25) $$(e_{L0}, f_{L0}, g_{L0}) = [(\hat{e}_{L0}, \hat{f}_{L0})\cos(n\pi y/\beta); \hat{g}_{L0}\sin(n\pi y/\beta)]\exp i(\alpha_L X + \sigma t)$$

with a similar expression for (e_{S0}, f_{S0}, g_{S0}). The latter satisfy (24*a, f*) provided the following dispersion relationships be satisfied:

(26) $$\alpha_L^2 = \sigma^2/h_L - n^2\pi^2/\beta^2,$$

(27) $$\alpha_S^2 = \sigma^2 - n^2\pi^2/\beta^2.$$

We then assume

(28) $$\zeta_0 = \hat{\zeta}_0 \exp(i\sigma t)\cos(n\pi y/\beta).$$

Imposing (24*g*) along with (24*a*) and the boundary conditions for f_{L0} and f_{S0} we end up with the following eigenrelationship for σ

(29) $$-m + k/\sigma^2 = h_L/\sqrt{n^2\pi^2/\beta^2 - \sigma^2/h_L} + 1/\sqrt{n^2\pi^2/\beta^2 - \sigma^2}$$

having assumed n to be large enough for α_L^2 and α_S^2 to be negative and having chosen the solution which decays as $X \to -\infty$ in the L-region and as $X \to \infty$ in the S-region.

Furthermore the constants $\hat{e}_{L0}, \hat{f}_{L0}, \hat{g}_{L0}, \hat{e}_{S0}, \hat{f}_{S0}, \hat{g}_{S0}$ are all expressed in terms of $\hat{\zeta}_0$ as follows:

$$(30) \qquad (\hat{e}_{L0}, \hat{f}_{L0}, \hat{g}_{L0}, \hat{e}_{S0}, \hat{f}_{S0}, \hat{g}_{S0}) =$$
$$= (-i(\sigma^2/\alpha_L), i\sigma, -(n\pi/\beta)(\sigma/\alpha_L), -2(\sigma^2/\alpha_S), i\sigma, -(n\pi/\beta)(\sigma/\alpha_S))\hat{\zeta}_0 .$$

5. Results

For given values of the dimensionless parameters m, k, h_L and β any given mode n is characterized by a «natural» frequency σ given by the solution of (29). We point out that the frequency ω^* of the incoming wave can be removed from (29) which can be written as a relationship between $(\beta\sigma)$, (m/β) and (βk), which are dimensionless quantities independent of ω^*.

It is then convenient to write (7*b*) and (9*a, b*) in the form

$$(31) \qquad m = \mathcal{M}/T, \qquad k = KT, \qquad \beta = b/T$$

where T is a dimensionless wave period defined as

$$(32) \qquad T = (2\pi/\omega^*)\sqrt{g/H_S^*} .$$

Using (31) the dispersion relationship (29) gives σ as a function of T for each mode n and given values of the parameters $\mathcal{M}, K, b$ and h_L.

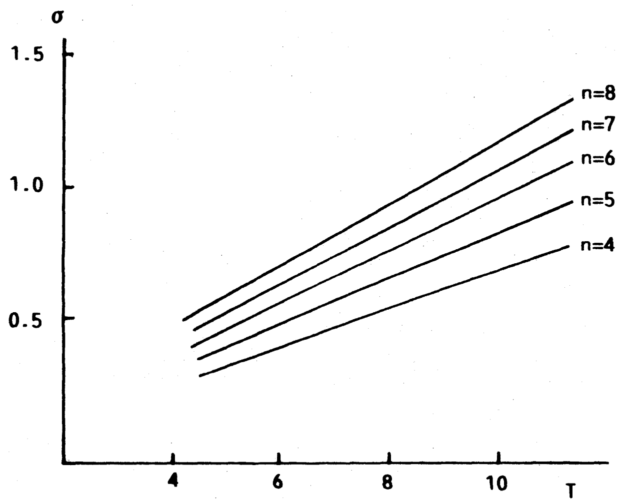

Fig. 2. – The frequency σ_n of the free modes n *versus* T for $\mathcal{M} = 2.0$, $K = 0.3$, $b = 160$, $h_L = 1$ and various n's.

A very rough calculation of the actual values of $\mathcal{M}$, K, b for the case of Malamocco inlet suggests the following estimate in the case $h_L = 1$

$$\mathcal{M} = 2.0\,, \qquad K = 0.3\,, \qquad b = 160\,. \tag{33}$$

Figure 2 shows the function $\sigma_n(T)$ for a few modes with the choice of $\mathcal{M}$ and K suggested by (33). Since typical values of T for large amplitude waves in the Adriatic sea close to the lagoon inlets range between 5 and 10 it appears that both subharmonic and synchronous responses are possible, the former occurring for relatively low values of both T and n while the latter may occur in the whole range of the feasible values of T for different values of n. In part 2 we investigate the conditions for the above subharmonic free modes to be excited by the incoming wave leaving the synchronous case for a future investigation.

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