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Angular limits and derivatives for holomorphic maps of infinite dimensional bounded homogeneous domains

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Geometria. — *Angular limits and derivatives for holomorphic maps of infinite dimensional bounded homogeneous domains.* Nota (*) di KAZIMIERZ WŁODARCZYK, presentata dal Socio E. Vesentini.

ABSTRACT. — An infinite dimensional extension of the Pick-Julia theorem is used to derive the conditions of Carathéodory type which guarantee the existence of angular limits and angular derivatives for holomorphic maps of infinite dimensional bounded symmetric homogeneous domains in J^*-algebras and in complex Hilbert spaces. The case of operator-valued analytic maps is considered and examples are given.

KEY WORDS: Infinite dimensional bounded symmetric homogeneous domain; Holomorphic map; Operator-valued analytic map; Angular derivative; J^*-algebra.

RIASSUNTO. — *Limiti e derivate angolari per le applicazioni olomorfe di domini limitati omogenei di dimensione infinita.* Da un'estensione di dimensione infinita del teorema di Pick-Julia vengono dedotte condizioni, «alla Carathéodory», sufficienti per l'esistenza di limiti angolari e derivate angolari per applicazioni olomorfe di domini limitati omogenei simmetrici in algebre J^* ed in spazi di Hilbert. Si considerano alcuni esempi e si studiano funzioni analitiche i cui valori sono degli operatori.

1. INTRODUCTION

The extensive investigations and applications of angular limits and angular derivatives to problems of function theory in finite dimensional complex spaces (in particular, to studies of the Wolff-Denjoy fixed points), initiated by Carathéodory, are well documented in a series of books and papers (see *e.g.* [1, 3-7, 10-12, 14, 16-19]). Rudin in [18] proved results for holomorphic maps of the unit balls in $\boldsymbol{C}^n$, analogous to the following classical theorem of Carathéodory.

THEOREM 1.1 [5, p. 96]. *Let* $\Delta = \{x \in \boldsymbol{C}\colon |x| < 1\}$ *be the open unit disc in the complex plane* $\boldsymbol{C}$ *and let* $F\colon \Delta \to \Delta$ *be a holomorphic map. If* $\{x_n\}$ *is any sequence of numbers lying within some angular set* $D_\alpha = \{x \in \boldsymbol{C}\colon |1 - x| < \alpha(1 - |x|^2)/2\}$, $\alpha > 1$, *and tending to* $x = 1$, *then* $\lim [1 - F(x_n)](1 - x_n)^{-1}$ *exists as* $n \to \infty$. *This limit is either* $+\infty$ *or a number* $L > 0$. *In the second case, we also have* $\lim DF(x_n) = L$ *as* $n \to \infty$, *and we refer to this number as the «angular derivative» of the map* F *at the point* $x = 1$.

Let H and K be Hilbert spaces over $\boldsymbol{C}$ and let $\mathcal{L}(H, K)$ denote the Banach space of all bounded linear operators X from H to K with the operator norm.

In infinite dimension, the situation is much more complicated and it is not easy to find formulations which are analogous to the finite variables results and have hopes to hold true. For a discussion of these problems, we refer to Ky Fan [9] where a nice generalization of Carathéodory's theorem for operator-valued

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analytic maps of the right half-plane $\Pi = \{x \in C: \operatorname{Re} x > 0\}$ into the Siegel domain $\{X \in \mathfrak{L}(H, H): \operatorname{Re} X > 0\}$ is given.

Let $\mathfrak{B} \subset \mathfrak{L}(H, K)$ denote a closed complex linear subspace of $\mathfrak{L}(H, K)$ such that $XX^*X \in \mathfrak{B}$ whenever $X \in \mathfrak{B}$, and let $\mathfrak{B}_0 = \{X \in \mathfrak{B}: \|X\| < 1\}$.

In this paper we use the ideas of functional analysis and operator theory to establish the conditions which guarantee the existence of angular limits and angular derivatives for holomorphic maps $F: \mathfrak{B}_0 \to \mathfrak{B}_0$ in bounded symmetric homogeneous domains $\mathfrak{B}_0$ of infinite dimensional complex Banach spaces $\mathfrak{B}$, called J^*-algebras, which were introduced and investigated by Harris in [13] (the set of the Harris domains includes the set of the classical Cartan [8] bounded symmetric homogeneous domains in C^n). The special case when J^*-algebra $\mathfrak{B}$ is a complex Hilbert space (*i.e.* when $\mathfrak{B} = K = \mathfrak{L}(C, K)$) is considered and, also, the conditions which guarantee the existence of angular limits and angular derivatives for operator-valued analytic maps of Δ into $\mathfrak{B}_0$ are given. We present three examples. This paper is a continuation of the studies in [25, 26].

2. Definitions, notations and statement of results

Let H and K be Hilbert spaces over C, let $\mathfrak{L}(H, K)$ denote the Banach space of all bounded linear operators X from H to K with the operator norm, and let $\mathfrak{B} \subset \mathfrak{L}(H, K)$ be a J^*-algebra, *i.e.* a closed complex linear subspace of $\mathfrak{L}(H, K)$ such that $XX^*X \in \mathfrak{B}$ whenever $X \in \mathfrak{B}$.

For two Hermitian operators $A, B \in \mathfrak{L}(H, H)$, we write $A \geqslant B$ to indicate that $A - B$ is a positive operator, *i.e.* $\langle (A - B)x, x\rangle \geqslant 0$ for all $x \in H$. The strict inequality $A > B$ means that $A - B$ is positive and invertible.

Let I_H and I_K denote the identity maps on H and K, respectively. If $X \in \mathfrak{L}(H, H)$, we write $\operatorname{Re} X = (X + X^*)/2$, $\operatorname{Im} X = (X - X^*)/(2i)$.

Let $\mathfrak{B}_0 = \{X \in \mathfrak{B}: \|X\| < 1\}$; for $Y \in \mathfrak{B}_0$, let $A_Y = I_H - Y^*Y$ and $B_Y = I_K - YY^*$, and let $T_Y: \mathfrak{B}_0 \to \mathfrak{B}_0$ denote the Möbius biholomorphic map of the form (see [13, Theorem 2, p. 20])

$$T_Y(X) = B_Y^{-1/2}(X - Y)(I_H - Y^*X)^{-1}A_Y^{1/2}, \qquad X \in \mathfrak{B}_0.$$

We start by proving the following general result of the Pick-Julia type in arbitrary J^*-algebras. It differs from those given in [2, 9, 22, 25].

THEOREM 2.1. *Let* $\mathfrak{B} \subset \mathfrak{L}(H, K)$ *be a* J^**-algebra. If* $F: \mathfrak{B}_0 \to \mathfrak{p}$, $\mathfrak{p} = \{X \in \mathfrak{L}(H, H): \operatorname{Re} X > 0\}$, *is a holomorphic map such that* $F(Z) = I_H$ *for some* $Z \in \mathfrak{B}_0$, *then*

$$\|F(X)\| \leqslant [1 + \|T_Z(X)\|][1 - \|T_Z(X)\|]^{-1} \quad \text{for } X \in \mathfrak{B}_0. \tag{2.1}$$

For $\alpha > 1$ and $Y \in \partial\mathfrak{B}_0$, we let

$$D_\alpha(Y) = \{X \in \mathfrak{B}: \|I_H - Y^*X\| < \alpha(1 - \|X\|^2)/2\}.$$

Of course, $D_\alpha(Y) \subset \mathfrak{B}_0$ for all $\alpha > 1$. When $\alpha \leqslant 1$ this set is empty. We call the sets $D_\alpha(Y)$, $\alpha > 1$, angular sets.

We shall need the following relation between $D_\alpha(Y)$ and $D_\beta(Y)$, $1 < \beta < \alpha$, $Y \in \mathfrak{B}_0$.

PROPOSITION 2.1. *Assume that* $Y \in \partial\mathfrak{B}_0$,

$$1 < \beta < \alpha, \quad \delta = (1/\beta - 1/\alpha)/3 \tag{2.2}$$

and $X \in D_\beta(Y)$, i.e.

$$\|I_H - Y^* X\| < \beta(1 - \|X\|^2)/2. \tag{2.3}$$

If

$$|\lambda| \leq \delta \|I_H - Y^* X\|, \tag{2.4}$$

then $X + \lambda Y \in D_\alpha(Y)$, i.e.

$$\|I_H - Y^*(X + \lambda Y)\| < \alpha(1 - \|X + \lambda Y\|^2)/2. \tag{2.5}$$

For $Y \in \partial\mathfrak{B}_0$, we define a holomorphic map $M_Y\colon \mathfrak{B}_0 \to \mathfrak{L}(H, H)$ by the formula $M_Y(X) = (I_H + Y^* X)(I_H - Y^* X)^{-1}$, $X \in \mathfrak{B}_0$. Let us observe that $\operatorname{Re} M_Y(X) = (I_H - X^* Y)^{-1}(I_H - X^* Y Y^* X)(I_H - Y^* X)^{-1}$, $X \in \mathfrak{B}_0$. Obviously, the operator $M_Y(X)$ is invertible, *i.e.* $[M_Y(X)]^{-1}$ exists and $M_Y(X) \in \mathfrak{p}$ for all $X \in \mathfrak{B}_0$ and $Y \in \partial\mathfrak{B}_0$.

The above results are the principal tools in the proof of the following theorem concerning the existence of angular limits and angular derivatives for holomorphic maps of bounded symmetric homogeneous domains in a J^*-algebra containing an isometry.

THEOREM 2.2. *Let* $\mathfrak{B} \subset \mathfrak{L}(H, K)$ *be a* J^**-algebra containing an isometry* U, *let* $F\colon \mathfrak{B}_0 \to \mathfrak{B}_0$ *be a holomorphic map in* $\mathfrak{B}_0$ *and let* $W \in \partial\mathfrak{B}_0$.

(*a*) *Suppose there is a Hermitian operator* $A \in \mathfrak{L}(H, H)$ *satisfying*

$$A^{1/2}[\operatorname{Re}(M_W \circ F)(X)]A^{1/2} > \operatorname{Re} M_U(X) \tag{2.6}$$

for all $X \in \mathfrak{B}_0$. *If* $D_\alpha(U)$ *stands for an angular set such that, for any* $\varepsilon > 0$, *there exists a point* $Z \in D_\alpha(U)$ *for which the inequality*

$$\|A^{1/2}[\operatorname{Re}(M_W \circ F)(Z)]A^{1/2} - \operatorname{Re} M_U(Z)\| < \varepsilon \tag{2.7}$$

holds, then

$$\lim \|[M_U(X)]A^{-1/2}[(M_W \circ F)(X)]^{-1} - A^{1/2}\| = 0 \tag{2.8}$$

and

$$\lim \|[\operatorname{Re} M_U(X)]A^{-1/2}[\operatorname{Re}(M_W \circ F)(X)]^{-1} - A^{1/2}\| = 0 \tag{2.9}$$

as $X \to U$, $X \in D_\alpha(U)$. *Moreover,*

$$\lim \|D\{A^{-1/2}[(M_W \circ F)(X)]^{-1} - [M_U(X)]^{-1}A^{1/2}\}(U)\| = 0 \tag{2.10}$$

as $X \to U$, $X \in D_\beta(U)$, $1 < \beta < \alpha$.

(*b*) *Suppose there is a Hermitian operator* $A \in \mathfrak{L}(H, H)$ *satisfying*

$$A^{1/2}[\operatorname{Re}(M_W \circ F)(X)]A^{1/2} = \operatorname{Re} M_U(X) \tag{2.11}$$

for all $X \in \mathfrak{B}_0$. *Then assertion* (2.8)-(2.10) *hold as* $X \to U$, $X \in D_\alpha(U)$, *for all* α, $1 < \alpha$.

REMARK 2.1. Equality (2.8) may be represented in the form $\lim (I_H - U^* X)^{-1} \cdot A^{-1/2}[I_H - W^* F(X)] = 2^{-1} A^{1/2}(I_H + W^* W)$ when $\lim F(X) = W$ as $X \to U$, $X \in D_\alpha(U)$. Moreover, since $D([M_U(X)]^{-1})(P) = -2(I_H + U^* X)^{-1} U^* P(I_H + U^* X)^{-1}$ and $D([(M_W \circ F)(X)]^{-1})(P) = -2[I_H + W^* F(X)]^{-1} W^* DF(X)(P)[I_H + W^* F(X)]^{-1}$, equality (2.10) may be represented in the form $\lim W^* DF(X)(U) = 4^{-1}(I_H + W^* W) \cdot A(I_H + W^* W)$ when $\lim F(X) = W$ as $X \to U$, $X \in D_\beta(U)$, $1 < \beta < \alpha$.

Let $K_0 = \{x \in K\colon \|x\| < 1\}$ and, for $y \in \partial K_0$, let $T_y\colon K_0 \to K_0$ denote the Möbius biholomorphic map of the form $T_y(x) = [E_y + (1 - \|y\|^2)^{1/2}(I_H - E_y)](x - y)(1 - \langle x, y\rangle)^{-1}$ where E_y denotes the linear projection of H onto the subspace $\{uy\colon u \in \boldsymbol{C}\}$.

For $y \in \partial K_0$, we define a holomorphic map $M_y\colon K_0 \to \boldsymbol{C} = \mathfrak{L}(\boldsymbol{C}, \boldsymbol{C})$ by the formula $M_y(x) = (1 + \langle x, y\rangle)(1 - \langle x, y\rangle)^{-1}$, $x \in K_0$. Let us observe that $\operatorname{Re} M_y(x) = (1 - |\langle x, y\rangle|^2)\,|1 - \langle x, y\rangle|^{-2}$, $x \in K_0$. Obviously, $M_y(x) \in \Pi$ for $x \in K_0$, where $\Pi = \{x \in \boldsymbol{C}\colon \operatorname{Re} x > 0\}$.

For $\alpha > 1$ and $y \in \partial K_0$, we let $D_\alpha(y) = \{x \in K\colon |1 - \langle x, y\rangle| < \alpha(1 - \|x\|^2)/2\}$.

If $H = \boldsymbol{C}$ and the J^*-algebra $\mathfrak{B}$ is of the form $\mathfrak{B} = K = \mathfrak{L}(\boldsymbol{C}, K)$, then Theorems 2.1 and 2.2 immediately yield the following two results.

THEOREM 2.3. *Let K be a complex Hilbert space. If $F\colon K_0 \to \Pi$ is a holomorphic map such that $F(z) = 1$ for some $z \in K_0$, then $|F(x)| \leq [1 + \|T_Z(x)\|][1 - \|T_Z(x)\|]^{-1}$ for $x \in K_0$.*

THEOREM 2.4. *Let K_0 be the open unit ball in a complex Hilbert space K, let $F\colon K_0 \to K_0$ be a holomorphic map in K_0 and let $u, w \in \partial K_0$.*

(*a*) *Suppose there is a number L satisfying $L[\operatorname{Re}(M_W \circ F)(x)] < \operatorname{Re} M_u(x)$ for all $x \in K_0$. If $D_\alpha(u)$ stands for an angular set such that, for any $\varepsilon > 0$, there exists a point $z \in D_\alpha(u)$ for which the inequality $L[\operatorname{Re}(M_W \circ F)(z)] - \operatorname{Re} M_u(z) < \varepsilon$ holds, then*

$$\lim |[M_u(x)][(M_W \circ F)(x)]^{-1} - L| = 0 \tag{2.12}$$

and

$$\lim |[\operatorname{Re} M_u(x)][\operatorname{Re}(M_W \circ F)(x)]^{-1} - L| = 0 \tag{2.13}$$

as $x \to u$, $x \in D_\alpha(u)$. Moreover,

$$\lim |D\{[(M_W \circ F)(x)]^{-1} - [M_u(x)]^{-1} L\}(u)| = 0 \tag{2.14}$$

as $x \to u$, $x \in D_\beta(u)$, $1 < \beta < \alpha$.

(*b*) *Suppose there is a number L satisfying $L[\operatorname{Re}(M_W \circ F)(x)] = \operatorname{Re} M_u(x)$ for all $x \in K_0$. Then assertions* (2.12)-(2.14) *hold as $x \to u$, $x \in D_\alpha(u)$ for all α, $1 < \alpha$.*

REMARK 2.2. Equalities (2.12) and (2.14) may be represented in the forms $\lim [1 - \langle F(x), w\rangle](1 - \langle x, u\rangle)^{-1} = L$ when $\lim F(x) = w$ as $x \to u$, $x \in D_\alpha(u)$, and $\lim \langle DF(x)(u), w\rangle = L$ when $\lim F(x) = w$ as $x \to u$, $x \in D_\beta(u)$, $1 < \beta < \alpha$, respectively.

Let $\Delta = \{x \in \boldsymbol{C}\colon |x| < 1\}$ and, for $y \in \partial\Delta$, let $T_y\colon \Delta \to \Delta$ denote the Möbius biholomorphic map of the form $T_y(x) = (x - y)(1 - \bar{y}x)^{-1}$.

For $y \in \partial\Delta$, we define a holomorphic map $M_y: \Delta \to C$ by the formula $M_y(x) = (1+\bar{y}x)(1-\bar{y}x)^{-1}$, $x \in \Delta$. Let us observe that $\operatorname{Re} M_y(x) = (1-|yx|^2)\cdot|1-\bar{y}x|^{-2} > 0$, $x \in \Delta$.

For $\alpha > 1$ and $y \in \partial\Delta$, we let

$$D_\alpha(y) = \{x \in C: |1-\bar{y}x| < \alpha(1-|x|^2)/2\}. \tag{2.15}$$

To continue, we require the analogue of Theorem 2.1 for operator-valued analytic maps. It takes the following form:

THEOREM 2.5. *If $F: \Delta \to \mathfrak{p}$ is an operator-valued analytic map in Δ such that $F(z) = I_H$ for some $z \in \Delta$, then $\|F(x)\| \leq [1+|T_z(x)|][1-|T_z(x)|]^{-1}$ for $x \in \Delta$.*

Theorem 2.2 remains valid for the operator-valued analytic maps announced before.

THEOREM 2.6. *Let $\mathfrak{B} \subset \mathcal{L}(H,K)$ be a J^*-algebra, let $F: \Delta \to \mathfrak{B}_0$ be an operator-valued analytic map in Δ and let $u \in \partial\Delta$, $W \in \partial\mathfrak{B}_0$.*

a) Suppose there is a Hermitian operator $A \in \mathcal{L}(H,H)$ satisfying $A^{1/2}[\operatorname{Re}(M_W \circ F)(x)]A^{1/2} > [\operatorname{Re} M_u(x)]I_H$ for all $x \in \Delta$. If $D_\alpha(u)$ stands for an angular set such that, for any $\varepsilon > 0$, there exists a point $z \in D_\alpha(u)$ for which the inequality $\|A^{1/2}[\operatorname{Re}(M_W \circ F)(z)]A^{1/2} - [\operatorname{Re} M_u(z)]I_H\| < \varepsilon$ holds, then

$$\lim \|[M_u(x)][(M_W \circ F)(x)]^{-1} - A\| = 0 \tag{2.16}$$

and

$$\lim \|[\operatorname{Re} M_u(x)][\operatorname{Re}(M_W \circ F)(x)]^{-1} - A\| = 0 \tag{2.17}$$

as $x \to u$, $x \in D_\alpha(u)$. Moreover,

$$\lim \|D\{[(M_W \circ F)(x)]^{-1} - [M_u(x)]^{-1}A\}(u)\| = 0 \tag{2.18}$$

as $x \to U$, $X \in D_\beta(u)$, $1 < \beta < \alpha$.

(b) Suppose there is a Hermitian operator $A \in \mathcal{L}(H,H)$ satisfying

$$A^{1/2}[\operatorname{Re}(M_W \circ F)(x)]A^{1/2} = [\operatorname{Re} M_u(x)]I_H \tag{2.19}$$

for all $x \in \Delta$. Then assertions (2.16)-(2.18) hold as $x \to u$, $x \in D_\alpha(u)$, for all α, $1 < \alpha$.

REMARK 2.3. Equalities (2.16) and (2.18) may be represented in the forms $\lim [I_H - W^*F(x)](1-\bar{u}x)^{-1} = 2^{-1}A(I_H + W^*W)$ when $\lim F(x) = W$ as $x \to u$, $x \in D_\alpha(u)$, and $\lim W^*DF(x)(u) = 4^{-1}(I_H + W^*W)A(I_H + W^*W)$ when $\lim F(x) = W$ as $x \to u$, $x \in D_\beta(u)$, $1 < \beta < \alpha$, respectively.

For a J^*-algebra $\mathfrak{B} \subset \mathcal{L}(H,K)$ containing an isometry U, we define the J^*-algebra $\mathfrak{B}_U = \{X_U \in \mathfrak{B}: X_U = xU, x \in C\} \subset \mathcal{L}(H,K)$ and, in $\mathfrak{B}_U$, the unit ball $(\mathfrak{B}_U)_0 = \{X_U \in \mathfrak{B}_U: X_U = xU, x \in \Delta\}$.

For $\alpha > 1$, an isometry $U \in \mathfrak{B}$ and $y \in \partial\Delta$, we let $D_\alpha(U,y) = \{X_U \in \mathfrak{B}_U: X_U = xU, x \in D_\alpha(y)\}$ where $D_\alpha(y)$ is defined by (2.15).

Theorem 2.1 also provides tools for proving the following analogue of Theorem 2.2:

THEOREM 2.7. *Let* $\mathfrak{B} \subset \mathcal{L}(H, K)$ *be a* J^**-algebra containing an isometry* U, *let* $F\colon \mathfrak{B}_0 \to \mathfrak{B}_0$ *be a holomorphic map in* $\mathfrak{B}_0$ *and let* $W \in \partial\mathfrak{B}_0$.

(*a*) *Suppose there is a Hermitian operator* $A \in \mathcal{L}(H, H)$ *satisfying* $A^{1/2}[\operatorname{Re}(M_W \circ F)(X_U)]A^{1/2} > \operatorname{Re} M_U(X_U)$ *for all* $X_U = xU \in (\mathfrak{B}_U)_0$. *If* $D_\alpha(U, 1)$ *stands for an angular set such that, for any* $\varepsilon > 0$, *there exists a point* $Z_U \in D_\alpha(U, 1)$ *for which the inequality* $\|A^{1/2}[\operatorname{Re}(M_W \circ F)(Z_U)]A^{1/2} - \operatorname{Re} M_U(Z_U)\| < \varepsilon$ *holds, then*

$$\lim \|[M_U(X_U)][(M_W \circ F)(X_U)]^{-1} - A\| = 0 \tag{2.20}$$

and

$$\lim \|[\operatorname{Re} M_U(X_U)][\operatorname{Re}(M_W \circ F)(X_U)]^{-1} - A\| = 0 \tag{2.21}$$

as $X_U \to U$, $X_U \in D_\alpha(U, 1)$. *Moreover,*

$$\lim \|D\{[(M_W \circ F)(X_U)]^{-1} - [M_U(X_U)]^{-1}A\}(U)\| = 0 \tag{2.22}$$

as $X_U \to U$, $X_U \in D_\beta(U, 1)$, $1 < \beta < \alpha$.

(*b*) *Suppose there is a Hermitian operator* $A \in \mathcal{L}(H, H)$ *satisfying*

$$A^{1/2}[\operatorname{Re}(M_W \circ F)(X_U)]A^{1/2} = \operatorname{Re} M_U(X_U)$$

for all $X_U \in (\mathfrak{B}_U)_0$. *Then assertions* (2.20)-(2.22) *hold as* $X_U \to U$, $X_U \in D_\alpha(U, 1)$, *for all* α, $1 < \alpha$.

REMARK 2.4. Equalities (2.20) and (2.22) may be represented in the forms $\lim [I_H - W^* F(X_U)](1 - x)^{-1} = 2^{-1}A(I_H + W^* W)$ when $\lim F(X_U) = W$ as $X_U \to U, X_U \in D_\alpha(U, 1)$, and $\lim W^* DF(X_U)(U) = 4^{-1}(I_H + W^* W)A(I_H + W^* W)$ when $\lim F(X_U) = W$ as $X_U \to U$, $X_U \in D_\beta(U, 1)$, $1 < \beta < \alpha$, respectively.

3. EXAMPLES

EXAMPLE 1. Let $\mathfrak{B} \subset \mathcal{L}(H, K)$ be a J^*-algebra containing an isometry U and let $F\colon \mathfrak{B}_0 \to \mathfrak{B}_0$ be a holomorphic map of the form $F(X) = (X + U)/2$. Then $F(U) = U$ and, for all $X \in \mathfrak{B}_0$,

$$\operatorname{Re}(M_U \circ F)(X) = (I_H - X^* U)^{-1}[2(I_H - X^* X) + (X^* - U^*)(X - U)](I_H - U^* X)^{-1};$$

$$\operatorname{Re} M_U(X) = (I_H - X^* U)^{-1}(I_H - X^* X)(I_H - U^* X)^{-1};$$

$$(I_H - U^* X)^{-1}[I_H - U^* F(X)] = 2^{-1}I_H, \qquad U^* DF(X)(U) = 2^{-1}I_H.$$

Consequently, the map F satisfies (2.6)-(2.10) for $W = U$ and for $A = 2^{-1}I_H$ in all angular sets $D_\alpha(U)$, $\alpha > 1$.

EXAMPLE 2. Let $\mathfrak{B} \subset \mathcal{L}(H, K)$ be a J^*-algebra containing a unitary operator U and let $F\colon \mathfrak{B}_0 \to \mathfrak{B}_0$ be a biholomorphic map in $\mathfrak{B}_0$ of the form (see [20, Section 3]) $F(X) = [aU + (2 - a)X][(2 + a)I_H - aU^* X]^{-1}$, $X \in \mathfrak{B}_0$, where $a \in \Pi$ is arbitrary and fixed. Then $F(U) = U$, $F(\mathfrak{B}_0) \neq \mathfrak{B}_0$ and, for $X \in \mathfrak{B}_0$, we have

$$\operatorname{Re}(M_U \circ F)(X) = (I_H - X^* U)^{-1}[I_H - X^*X + (\operatorname{Re} a)(X^* - U^*)(X - U)](I_H - U^*X)^{-1};$$

$$\operatorname{Re} M_U(X) = (I_H - X^* U)^{-1}(I_H - X^* X)(I_H - U^* X)^{-1};$$

$$(I_H - U^* X)^{-1}[I_H - U^* F(X)] = 2[(2 + a) I_H - aU^* X]^{-1};$$

$$U^* DF(X)(U) = 4U^*[(2 + a) I_K - aXU^*]^{-1} U[(2 + a) I_H - aU^* X]^{-1}.$$

Consequently, the map F satisfies (2.6)-(2.10) for $W = U$ and for $A = I_H$ in all angular sets $D_\alpha(U)$, $\alpha > 1$.

EXAMPLE 3. Let $\mathfrak{B} \subset \mathfrak{L}(H, K)$ be a J^*-algebra containing an isometry V and let $0 < |a| < 1$ be arbitrary and fixed. Let $F = T_{aV}$. Then F is a biholomorphic map of $\mathfrak{B}_0$ onto $\mathfrak{B}_0$, has two fixed points $U_1 = a|a|^{-1}V$ and $U_2 = -a|a|^{-1}V$ and is of the form (see [20, Theorem 2.1 (c)])

$$F(X) = (I_K - |a|^2 VV^*)^{-1/2}(X - aV)(I_H - \bar{a}V^* X)^{-1}(1 - |a|^2)^{1/2}, \qquad X \in \mathfrak{B}_0.$$

For $X \in \mathfrak{B}_0$ and for $i = 1, 2$, we have

$$\operatorname{Re} M_{U_i}(X) = (I_H - X^* U_i)^{-1}(I_H - X^* VV^* X)(I_H - U_i^* X)^{-1}.$$

Further, since (see [23, formula (7), p. 328])

$$V^*(I_K - |a|^2 VV^*)^{-1/2} = (I_H - |a|^2 V^* V)^{-1/2} V^*,$$

we obtain $F(X)^* V = (I_H - aX^* V)^{-1}(X^* V - \bar{a} I_H)$, $X \in \mathfrak{B}_0$. Consequently, if we denote $A_1 = (1 + |a|)(1 - |a|)^{-1} I_H$ and $A_2 = (1 - |a|)(1 + |a|)^{-1} I_H$, then, for $X \in \mathfrak{B}_0$, we get

$$A_i[\operatorname{Re}(M_{W_i} \circ F)(X)] = \operatorname{Re} M_{U_i}(X) \text{ for } W_i = U_i, \qquad i = 1, 2;$$

$$(I_H - U_1^* X)^{-1}[I_H - U_1^* F(X)] = (1 + |a|)(I_H - \bar{a}V^* X)^{-1};$$

$$(I_H - U_2^* X)^{-1}[I_H - U_2^* F(X)] = (1 - |a|)(I_H + \bar{a}V^* X)^{-1}$$

and

$$U_i^* DF(X)(U_i) = U_i^*(I_K - |a|^2 VV^*)^{1/2}(I_K - \bar{a}XV^*)^{-1} U_i (I_H - \bar{a}V^* X)^{-1}(1 - |a|^2)^{1/2}.$$

Hence all the assumptions and assertions of Theorem 2.2 (b) are satisfied.

REMARK 3.1. In Examples 1-3 the maps F and their fixed points satisfy the assumptions of [21, Lemma 2.1].

4. PROOF OF THEOREM 2.1

Let $\mathfrak{D}_0 = \{X \in \mathfrak{L}(H, H): \|X\| < 1\}$ and let $f: \mathfrak{D}_0 \to \mathfrak{p}$ be a Cayley biholomorphic map of the form $f(X) = (I_H + X)(I_H - X)^{-1}$, $X \in \mathfrak{D}_0$.

We define a map $g: \mathfrak{B}_0 \to \mathfrak{D}_0$ by the formula $g(X) = (f^{-1} \circ F \circ T_Z^{-1})(X)$, $X \in \mathfrak{B}_0$. Since $g(0) = (f^{-1} \circ F)(Z) = f^{-1}(I_H) = 0$, using Schwarz's lemma, we obtain $\|g(S)\| \leq \|S\|$ for $S \in \mathfrak{B}_0$. In particular, for $S = T_Z(X)$, we get $\|(f^{-1} \circ F)(X)\| \leq$ $\leq \|T_Z(X)\|$, $X \in \mathfrak{B}_0$, or, equivalently, $\{(f^{-1} \circ F)(X)\}^* \{(f^{-1} \circ F)(X)\} \leq r^2 I_H$ where $r = \|T_Z(X)\|$. But $(f^{-1} \circ F)(X) = [F(X) - I_H][I_H + F(X)]^{-1}$, and thus, $F(X)^* F(X) -$

$-(1+r)(1-r)^{-1}F(X)^* - (1+r)(1-r)^{-1}F(X) + I_H \leqslant 0$. Consequently, $\|F(X) - (1+r)(1-r)^{-1}I_H\| \leqslant 2r^{1/2}(1-r)^{-1}$, which yields (2.1).

5. Proof of Proposition 2.1

From (2.2) we have

$$(5.1) \qquad |\lambda|^2 < |\lambda|, \quad 2/\alpha < 2, \quad (5\delta + 2/\alpha) < 2/\beta$$

whenever $|\lambda|$ is sufficiently small. From (2.3) we get

$$(5.2) \qquad \|X\|^2 + (2/\beta)\|I_H - Y^*X\| < 1.$$

Thus, using (5.1), (2.4) and (5.2), we obtain

$$\|X+\lambda Y\|^2 + (2/\alpha)\|I_H - Y^*(X+\lambda Y)\| \leqslant \|X\|^2 + 3|\lambda| + (2/\alpha)\|I_H - Y^*X\| + $$
$$+(2/\alpha)|\lambda| \leqslant \|X\|^2 + 5|\lambda| + (2/\alpha)\|I_H - Y^*X\| \leqslant \|X\|^2 + (2/\beta)\|I_H - Y^*X\| \leqslant 1.$$

This yields (2.5).

6. Proof of Theorem 2.2

(*a*) Let $\varepsilon > 0$ be arbitrary and fixed. By (2.7), there exists $Z \in D_\alpha(U)$ such that $\|A^{1/2}[\mathrm{Re}\,(M_W \circ F)(Z)]A^{1/2} - \mathrm{Re}\, M_U(Z)\| < \varepsilon$. We define maps E and G, holomorphic in $\mathfrak{B}_0$, by the formulae $E(X) = A^{1/2}[(M_W \circ F)(X)]A^{1/2} - M_U(X)$ and

$$(6.1) \qquad G(X) = [\mathrm{Re}\, E(Z)]^{-1/2}[E(X) - i \cdot \mathrm{Im}\, E(Z)][\mathrm{Re}\, E(Z)]^{-1/2},$$

respectively. Let us observe that, by (2.6), $\mathrm{Re}\, E(X) > 0$ and $\mathrm{Re}\, G(X) > 0$ for all $X \in \mathfrak{B}_0$, $G(Z) = I_H$ and, since (see [24, formula (18), p. 247]) $(1 - \|T_Z(X)\|^2)^{-1} = D(Z,X)$ where $D(Z,X) = \|A_Z^{-1/2}(I_H - Z^*X)A_X^{-1}(I_H - X^*Z)A_Z^{-1/2}\|$, applying Theorem 2.1 to the map G, we get $\|G(X)\| \leqslant [1 + (1 - D(Z,X)^{-1})^{1/2}][1 - (1 - D(Z,X)^{-1})^{1/2}]^{-1} = D(Z,X)[1 + \|T_Z(X)\|]^2 < 4D(Z,X)$ for $X \in \mathfrak{B}_0$. Now, from (6.1) we obtain

$$\|E(X)[M_U(X)]^{-1}\| = \|A^{1/2}[(M_W \circ F)(X)]A^{1/2}[M_U(X)]^{-1} - I_H\| \leqslant$$
$$\leqslant \|[M_U(X)]^{-1}\| \|[\mathrm{Re}\, E(Z)]^{1/2}G(X)[\mathrm{Re}\, E(Z)]^{1/2} + i \cdot \mathrm{Im}\, E(Z)\| \leqslant$$
$$\leqslant \|[M_U(X)]^{-1}\| \{\|\mathrm{Re}\, E(Z)\| \cdot \|G(X)\| + \|\mathrm{Im}\, E(Z)\|\}.$$

Since $\|M_U(X)]^{-1}\| \leqslant \|I_H - U^*X\| \|[I_H + U^*X]^{-1}\|$ and

$$D(Z,X) \leqslant \|I_H - X^*Z\|^2[(1 - \|X\|^2)(1 - \|Z\|^2)]^{-1},$$

it follows that

$$\|A^{1/2}[(M_W \circ F)(X)]A^{1/2}[M_U(X)]^{-1} - I_H\| \leqslant 2\varepsilon\alpha \cdot (1 - \|Z\|^2)^{-1}\|[I_H + U^*X]^{-1}\| \cdot$$
$$\cdot \|I_H - X^*Z\|^2 + (\alpha/2)(1 - \|X\|^2)\|[I_H + U^*X]^{-1}\| \|\mathrm{Im}\, E(Z)\|.$$

Moreover, since the right-hand side of the above inequality tends to $\varepsilon\alpha(1 - \|Z\|^2)^{-1}\|I_H - U^*Z\|^2 < \varepsilon\alpha^2$ and, since $\varepsilon > 0$ can be arbitrarily small, this proves that $\lim \|A^{-1/2}[M_U(X)]A^{-1/2}[(M_W \circ F)(X)]^{-1} - I_H\| = 0$ as $X \to U$, $X \in D_\alpha(U)$, *i.e.* (2.8) holds.

Now, let us observe that

$$\|[\mathrm{Re}\, E(X)][\mathrm{Re}\, M_U(X)]^{-1}\| = \|A^{1/2}[\mathrm{Re}\,(M_W\circ F)(X)]A^{1/2}[\mathrm{Re}\, M_U(X)]^{-1} - I_H\| \leqslant$$
$$\leqslant \|E(X)\|\,\|[\mathrm{Re}\, M_U(X)]^{-1}\| \leqslant \|E(X)[M_U(X)]^{-1}\|\,\|M_U(X)\|\,\|[\mathrm{Re}\, M_U(X)]^{-1}\|.$$

But

$$\|M_U(X)\| \leqslant \|I_H + U^*X\|(1-\|X\|)^{-1}$$

and

$$\|[\mathrm{Re}\, M_U(X)]^{-1}\| \leqslant \|I_H - U^*X\|^2(1-\|X\|^2)^{-1}.$$

Thus $\|A^{1/2}[\mathrm{Re}\,(M_W\circ F)(X)]A^{1/2}[\mathrm{Re}\, M_U(X)]^{-1} - I_H\| \leqslant \alpha^2\|E(X)[M_U(X)]^{-1}\|$. Since, by (2.8), the right-hand side of the above inequality tends to zero, this proves that $\lim \|A^{-1/2}[\mathrm{Re}\, M_U(X)]A^{-1/2}[\mathrm{Re}\,(M_W\circ F)(X)]^{-1} - I_H\| = 0$ as $X\to U$, $X\in D_\alpha(U)$, *i.e.* (2.9) holds.

Now, we prove (2.10). By Proposition 2.1 and the Cauchy integral formula ([15, Proposition 2, p. 21]), we have

$$D\{A^{-1/2}[(M_W\circ F)(X)]^{-1} - [M_U(X)]^{-1}A^{1/2}\}(U) = \frac{1}{2\pi i}\cdot \tag{6.2}$$
$$\cdot \int_{|\lambda|=r} \frac{[M_U(X+\lambda U)]^{-1}}{\lambda^2}\{[M_U(X+\lambda U)]A^{-1/2}[(M_W\circ F)(X+\lambda U)]^{-1} - A^{1/2}\}\,d\lambda$$

where $\lambda = r\cdot\exp(it)$, $r = r(X) = \delta\|I_H - U^*X\|$, $t\in[0;2\pi]$. But

$$\|[M_U(X+\lambda U)]^{-1}\|\,|\lambda|^{-1} \leqslant [\|I_H - U^*X\| + |\lambda|]\,\|[I_H + U^*(X+\lambda U)]^{-1}\|\,|\lambda|^{-1} =$$
$$= (\delta^{-1}+1)\,\|[I_H + U^*(X+\lambda U)]^{-1}\|.$$

Since the right-hand side of the above inequality tends to $2^{-1}(\delta^{-1}+1)$, from (6.2), using (2.8) and Proposition 2.1, we get (2.10).

(*b*) If (2.11) holds for all $X\in\mathfrak{B}_0$, let $\varepsilon>0$ be arbitrary and fixed and let δ be such that $0<\delta<\varepsilon$. Then $A^{1/2}[\mathrm{Re}\,(M_W\circ F)(X)]A^{1/2} + \delta I_H > \mathrm{Re}\, M_U(X)$ for all $X\in \mathfrak{B}_0$. Moreover, obviously, then there exists some $Z\in D_\alpha(U)$ for which the inequality $\|A^{1/2}[\mathrm{Re}\,(M_W\circ F)(Z)]A^{1/2} - \mathrm{Re}\, M_U(Z) + \delta I_H\| = \delta < \varepsilon$ holds. Now, we define maps E_δ and G_δ, holomorphic in $\mathfrak{B}_0$, by the formulae $E_\delta(X) = E(X) + \delta I_H$, $E(X) = A^{1/2}[(M_W\circ F)(X)]A^{1/2} - M_U(X)$ and $G_\delta(X) = [\mathrm{Re}\, E_\delta(Z)]^{-1/2}[E_\delta(X) - i\cdot \mathrm{Im}\, E_\delta(Z)][\mathrm{Re}\, E_\delta(Z)]^{-1/2}$, respectively. Let us observe that $\mathrm{Re}\, E_\delta(X)>0$ and $\mathrm{Re}\, G_\delta(X)>0$ for all $X\in\mathfrak{B}_0$, $G_\delta(Z) = I_H$ and, using analogous considerations as in part (*a*), we have, respectively,

$$\|E(X)[M_U(X)]^{-1}\| = \|A^{1/2}[(M_W\circ F)(X)]A^{1/2}[M_U(X)]^{-1} - I_H\| \leqslant$$
$$\leqslant 2\varepsilon\alpha(1-\|Z\|^2)^{-1}\,\|[I_H + U^*X]^{-1}\|\,\|I_H - X^*Z\|^2 +$$
$$+(\alpha/2)(1-\|X\|^2)\,\|[I_H+U^*X]^{-1}\|\,\{\|\mathrm{Im}\, E(Z)\| + \delta\}.$$

This implies (2.8). Moreover, using analogous argumentation as in part (*a*), we prove that also (2.9) and (2.10) hold as $X\to U$, $X\in D_\alpha(U)$, for all α, $1<\alpha$.

7. Proof of Theorem 2.6

(*a*) Let $\varepsilon > 0$ be arbitrary and fixed. By our assumptions, there exists $z \in D_\alpha(u)$ such that $\|A^{1/2}[\mathrm{Re}\,(M_W \circ F)(z)]A^{1/2} - [\mathrm{Re}\, M_u(z)]I_H\| < \varepsilon$.

We define operator-valued maps E and G, analytic in Δ, by the formulae $E(x) = A^{1/2}[(M_W \circ F)(x)]A^{1/2} - [M_u(x)]I_H$ and

$$G(x) = [\mathrm{Re}\, E(z)]^{-1/2}[E(x) - i\cdot \mathrm{Im}\, E(z)]\,[\mathrm{Re}\, E(z)]^{-1/2}\,, \tag{7.1}$$

respectively. Let us observe that $\mathrm{Re}\; E(x) > 0$ and $\mathrm{Re}\; G(x) > 0$ for all $x \in \Delta$, $G(z) = I_H$ and, since $[1 - |(x-z)(1-\bar{z}x)^{-1}|^2]^{-1} = d(z,x)$ where $d(z,x) = |1-\bar{z}x|^2 \cdot$ $\cdot[(1-|z|^2)(1-|x|^2)]^{-1}$ applying Theorem 2.5 to the map G, we get

$$\|G(x)\| \leqslant d(z,x)\cdot[1 + |(x-z)(1-\bar{z}x)^{-1}|]^2 < 4d(z,x) \tag{7.2}$$

for $x \in \Delta$. Now, from (7.1), we obtain

$$\|E(x)[M_u(x)]^{-1}\| = \|A^{1/2}[(M_W\circ F)(x)]A^{1/2}[M_u(x)]^{-1} - I_H\| \leqslant$$
$$\leqslant |[M_u(x)]^{-1}|\{\|\mathrm{Re}\; E(z)\|\cdot\|G(x)\| + \|\mathrm{Im}\; E(z)\|\}\,.$$

Consequently, by (7.2),

$$\|A^{1/2}[(M_W\circ F)(x)]A^{1/2}[M_u(x)]^{-1} - I_H\| \leqslant$$
$$\leqslant 2\varepsilon\alpha\,|1-\bar{z}x|^2[\,|1+\bar{u}x|\,(1-|z|^2)]^{-1} + (\alpha/2)(1-|x|^2)\,|1+\bar{u}x|^{-1}\,\|\mathrm{Im}\; E(z)\|\,.$$

Since the right-hand side of the above inequality tends to $\varepsilon\alpha(1-|z|^2)^{-1}|1 -$ $-\bar{z}u|^2 < \varepsilon\alpha^2$ and, since $\varepsilon > 0$ can be arbitrarily small, this proves that $\lim \|[M_u(x)][(M_W\circ F)(x)]^{-1} - A\| = 0$ as $x \to u$, $x \in D_\alpha(u)$, *i.e.* (2.16) holds.

Now, let us observe that

$$\|[\mathrm{Re}\; E(x)][\mathrm{Re}\; M_u(x)]^{-1}\| = \|A^{1/2}[\mathrm{Re}\,(M_W\circ F)(x)]A^{1/2}[\mathrm{Re}\; M_u(x)]^{-1} - I_H\| \leqslant$$
$$\leqslant \|E(x)\|\,|[\mathrm{Re}\; M_u(x)]^{-1}| \leqslant \|E(x)[M_u(x)]^{-1}\|\,|M_u(x)|\,|[\mathrm{Re}\; M_u(x)]^{-1}|\,.$$

But

$$|M_u(x)| \leqslant |1+\bar{u}x|\,(1-|x|)^{-1}$$

and

$$|[\mathrm{Re}\; M_u(x)]^{-1}| \leqslant |1-\bar{u}x|^2(1-|x|^2)^{-1}\,.$$

Thus $\|A^{1/2}[\mathrm{Re}\,(M_W\circ F)(x)]A^{1/2}[\mathrm{Re}\; M_u(x)]^{-1} - I_H\| \leqslant \alpha^2\|E(x)[M_u(x)]^{-1}\|$. Since the right-hand side of the above inequality tends to zero, this proves that $\lim \|[\mathrm{Re}\; M_u(x)][\mathrm{Re}\,(M_W\circ F)(x)]^{-1} - A\| = 0$ as $x \to u$, $x \in D_\alpha(u)$, *i.e.* (2.17) holds.

Using (2.19), we prove equality (2.18) analogously as equality (2.10).

(*b*) We use analogous considerations as in the proof of Theorem 2.2 (*b*).

References

[1] L. V. Ahlfors, *Conformal invariants: topics in geometric function theory.* McGraw-Hill, New York 1973.

[2] T. Ando - Ky Fan, *Pick-Julia theorems for operators.* Math. Z., 168, 1979, 23-34.

[3] K. F. Barth - P. J. Rippon - L. S. Sons, *Angular limits of holomorphic and meromorphic functions.* J. London Math. Soc., (2), 42, 1990, 279-291.

[4] R. D. BERMAN, *Angular limits and infinite asymptotic values of analytic functions of slow growth.* Ill. J. Math., 34, 1990, 845-858.
[5] C. CARATHÉODORY, *Conformal representations.* Cambridge Tracts in Mathematics and Mathematical Physics, Cambridge 1952.
[6] C. CARATHÉODORY, *Über die Winkelderivierten von beschrankten analytischen Functionen.* Sitz. Ber. Preuss. Akad., Phys.-Math., IV, 1929, 1-18.
[7] C. CARATHÉODORY, *Theory of functions.* Vol. 2. Chelsea Publishing Company, New York 1960.
[8] E. CARTAN, *Sur les domaines bornés homogènees de l'espace de n variables complexes.* Abh. Math. Sem. Univ. Hamburg, 11, 1935, 116-162.
[9] KY FAN, *The angular derivative of an operator-valued analytic function.* Pacific J. Math., 121, 1986, 67-72.
[10] S. J. GARDINER, *Angular limits of holomorphic functions which satisfy an integrability condition.* Monatsh. Math., 114, 1992, 97-106.
[11] D. GIRELA, *Non-tangential limits for analytic functions of slow growth in a disc.* J. London Math. Soc., (2), 46, 1992, 140-148.
[12] J. L. GOLDBERG, *Functions with positive real part in a half plane.* Duke Math. J., 29, 1962, 333-339.
[13] L. A. HARRIS, *Bounded symmetric homogeneous domains in infinite dimensional spaces.* Lecture Notes in Mathematics, 364, Springer-Verlag, Berlin-Heidelberg-New York 1974, 13-40.
[14] B. D. MACCLUER - J. H. SHAPIRO, *Angular derivatives and compact composition operators on the Hardy and Bergman spaces.* Canadian J. Math., 38, 1986, 878-906.
[15] L. NACHBIN, *Topology on spaces of holomorphic mappings.* Springer-Verlag, Berlin-Heidelberg-New York 1969.
[16] R. NEVANLINNA, *Analytic functions.* Springer-Verlag, Berlin-Heidelberg-New York 1970.
[17] CH. POMMERENKE, *Univalent functions.* Vandenhoeck and Ruprecht, Göttingen 1975.
[18] W. RUDIN, *Function theory in the unit ball of* C^n. Springer-Verlag, New York-Heidelberg-Berlin 1980.
[19] D. SARASON, *Angular derivatives via Hilbert space.* Complex Variables Theory Appl., 10, 1988, 1-10.
[20] K. WŁODARCZYK, *Hyperbolic geometry in bounded symmetric homogeneous domains of* J^**-algebras.* Atti Sem. Mat. Fis. Univ. Modena, 39, 1991, 201-211.
[21] K. WŁODARCZYK, *Julia's lemma and Wolff's theorem for* J^**-algebras.* Proc. Amer. Math. Soc., 99, 1987, 472-476.
[22] K. WŁODARCZYK, *Pick-Julia theorems for holomorphic maps in* J^**-algebras and Hilbert spaces.* J. Math. Anal. Appl., 120, 1986, 567-571.
[23] K. WŁODARCZYK, *Some properties of analytic maps of operators in* J^**-algebras.* Monatsh. Math., 96, 1983, 325-330.
[24] K. WŁODARCZYK, *Studies of iterations of holomorphic maps in* J^**-algebras and complex Banach spaces.* Quart. J. Math. Oxford, 37, 1986, 245-256.
[25] K. WŁODARCZYK, *The angular derivative of Fréchet-holomorphic maps in* J^**-algebras and complex Hilbert spaces.* Proc. Kon. Nederl. Akad. Wetensch., A91, 1988, 455-468; Indag. Math., 50, 1988, 455-468.
[26] K. WŁODARCZYK, *The Julia-Carathéodory theorem for distance-decreasing maps on infinite dimensional hyperbolic spaces.* Rend. Mat. Acc. Lincei, s. 9, v. 4, 1993, 171-179.

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