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Existence and continuous dependence results in the dynamical theory of piezoelectricity

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Fisica matematica. — *Existence and continuous dependence results in the dynamical theory of piezoelectricity.* Nota di MICHELE CIARLETTA, presentata (*) dal Socio T. Manacorda.

ABSTRACT. — The paper is concerned with the dynamical theory of linear piezoelectricity. First, an existence theorem is derived. Then, the continuous dependence of the solutions upon the initial data and body forces is investigated.

KEY WORDS: Piezoelectricity; Dynamical; Existence; Stability.

RIASSUNTO. — *Teoremi di esistenza e dipendenza continua nella dinamica dei materiali piezoelettrici.* Nell'ambito della teoria lineare dei processi dinamici dei materiali piezoelettrici, si studiano teoremi di esistenza e di dipendenza continua.

1. INTRODUCTION

The interaction of electromagnetic fields with deformable bodies has been the subject of many investigations. Extensive reviews in this direction can be found in the works of Dökmeci [1], Nowacki [2], Toupin [3], Eringen and Dixon [4], Parkus [5], Grot [6], Maugin [7].

This paper is concerned with the dynamical theory of linear piezoelectricity with dissipative boundary conditions. Existence results in the static theory of linear piezoelectricity have been established in [8]. Uniqueness results and minimum principles in the dynamical theory of piezoelectricity have derived in [9]. In the first part of the paper we use the results of the semigroups theory of linear operators to obtain an existence theorem. Dafermos [10] and Navarro and Quintanilla [11] have used a similar method to study boundary-initial-value problems in other theories of continuum mechanics. Then we investigate the continuous dependence of solutions upon the initial data and body forces.

2. BASIC EQUATIONS

We consider a body that at time $t = 0$ occupies the region B of Euclidean three-dimensional space and is bounded by the piecewise smooth surface ∂B. The motion of the body is referred to the reference configuration B and the fixed system of rectangular Cartesian axes Ox_k $(k = 1, 2, 3)$. We shall employ the usual summation and differentiation conventions: Latin subscripts are understood to range over the integers $(1, 2, 3)$, summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate. In all that follows, we use a superposed dot to denote partial differentiation with respect to the time. Letters in boldface stand for tensors of an order $p \geqslant 1$ and if $\boldsymbol{v}$ has

(*) Nella seduta del 16 dicembre 1995.

the order p, we write $v_{ij\ldots k}$ ($p$subscripts) for the components of $\boldsymbol{v}$ in the underlying rectangular Cartesian coordinate frame.

In this paper we consider the dynamical theory of linear piezoelectricity (see, *e.g.* [3-6]). The Maxwell equations have the form

$$\varepsilon_{irs}e_{s,r} + (1/c)\dot{b}_i = 0\,, \qquad \varepsilon_{irs}b_{s,r} - (1/c)\dot{d}_i = 0\,, \tag{2.1}$$

$$b_{i,i} = 0\,, \qquad d_{i,i} = 0\,, \tag{2.2}$$

on $B \times (0, t_1)$. Here $\boldsymbol{e}$ is the electric field, $\boldsymbol{b}$ is the magnetic flux density, $\boldsymbol{d}$ is the electric displacement, c is the velocity of light in vacuum, ε_{irs} is the alternating symbol, and t_1 is some finite time instant.

The equations of motion are given by

$$t_{ji,j} + \varrho f_i = \varrho \ddot{u}_i\,, \tag{2.3}$$

on $B \times (0, t_1)$, where t_{ij} is the stress tensor, $\boldsymbol{f}$ is the body force per unit mass, ϱ is the mass density, and $\boldsymbol{u}$ is the displacement vector field. The strain field associated with $\boldsymbol{u}$ is defined by

$$E_{ij} = (u_{i,j} + u_{j,i})/2\,. \tag{2.4}$$

Throughout this paper we assume that the body is homogeneous. The constitutive equations are given by

$$t_{ij} = C_{ijrs}E_{rs} - D_{kij}e_k\,, \qquad d_i = D_{irs}E_{rs} + \Gamma_{ij}e_j\,, \tag{2.5}$$

where C_{ijrs}, D_{mij} and Γ_{ij} are characteristic constants of the material. The coefficients C_{ijrs}, D_{mij} and Γ_{ij} have the following symmetries

$$C_{ijrs} = C_{rsij} = C_{jirs}\,, \qquad D_{mij} = D_{mji}\,, \qquad \Gamma_{ij} = \Gamma_{ji}\,. \tag{2.6}$$

To the system of field equations we adjoin the initial conditions

$$\begin{cases} u_i(\boldsymbol{x}, 0) = \hat{u}_i(\boldsymbol{x})\,, & \dot{u}_i(\boldsymbol{x}, 0) = \hat{v}_i(\boldsymbol{x})\,, \\ b_i(\boldsymbol{x}, 0) = \hat{b}_i(\boldsymbol{x})\,, & d_i(\boldsymbol{x}, 0) = \hat{d}_i(\boldsymbol{x})\,, \qquad \boldsymbol{x} \in \overline{B}, \end{cases} \tag{2.7}$$

where $\hat{u}_i$, $\hat{v}_i$, $\hat{b}_i$ and $\hat{d}_i$ are prescribed functions. We assume that

$$\hat{b}_{i,i} = 0\,, \qquad \hat{d}_{i,i} = 0 \ \text{ on } \overline{B}\,. \tag{2.8}$$

We note that the equations (2.2) are immediate consequences of the equations (2.1) and (2.8). The equations (2.1), (2.3), (2.4) and (2.5), if Γ_{ij} is invertible, combine to yield the following system

$$\begin{cases} \varrho\ddot{u}_i = C_{ijrs}u_{r,sj} - D_{mij}e_{m,j} + \varrho f_i\,, \\ \dot{e}_i = \chi_{ij}(c\varepsilon_{jrs}b_{s,r} - D_{jrs}\dot{u}_{r,s})\,, \\ \dot{b}_i = -c\varepsilon_{irs}e_{s,r}\,. \end{cases} \tag{2.9}$$

Here χ_{ij} is defined by

$$\Gamma_{ir}\chi_{rj} = \delta_{ij}\,, \tag{2.10}$$

where δ_{ij} is the Kronecker delta.

The conditions (2.7) may be written in the form

$$\boldsymbol{u} = \widehat{\boldsymbol{u}}, \quad \dot{\boldsymbol{u}} = \widehat{\boldsymbol{v}}, \quad \boldsymbol{e} = \widehat{\boldsymbol{e}}, \quad \boldsymbol{b} = \widehat{\boldsymbol{b}} \quad \text{on } \overline{B}, \tag{2.11}$$

where

$$\widehat{e}_i = \chi_{ij}(\widehat{d}_j - D_{jrs}\widehat{u}_{r,s}).$$

We consider the following boundary conditions

$$\begin{cases} e_i = \delta\varepsilon_{irs}b_r n_s & \text{on } \partial B \times [0, t_1], \\ u_i = 0 & \text{on } \overline{S}_1 \times [0, t_1], \\ v_i = -\nu t_{ij}n_j & \text{on } S_2 \times [0, t_1], \end{cases} \tag{2.12}$$

where δ, ν are positive coefficients characteristic of the boundary, while S_1 and S_2 are subsets of ∂B such that $\overline{S}_1 \cup S_2 = \partial B$, $S_1 \cap S_2 = \emptyset$, and $\boldsymbol{n}$ is the outward unit normal of ∂B.

3. Existence and continuous dependence results

Throughout this section we assume that:

(i) the density ϱ is strictly positive;

(ii) C_{ijrs} and Γ_{ij} are positive definite, *i.e.* there exist the positive constants c_0 and ε_0 such that

$$C_{ijrs}\xi_{ij}\xi_{rs} \geqslant c_0\xi_{ij}\xi_{ij}, \qquad \Gamma_{ij}\eta_i\eta_j \geqslant \varepsilon_0\eta_i\eta_i, \tag{3.1}$$

for every symmetric tensor ξ_{ij} and every vector η_i. In the first part of this section we use results of the semigroups theory of linear operators to obtain an existence theorem. Recently, Navarro and Quintanilla [11] have used this method to obtain existence results in thermoelasticity.

Let

$$X = \{w = (\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{e}, \boldsymbol{b});\ \boldsymbol{u} \in \boldsymbol{H}_0^1(B), \boldsymbol{v} \in \boldsymbol{H}^0(B), \boldsymbol{e} \in \boldsymbol{H}^0(B), \boldsymbol{b} \in \boldsymbol{H}^0(B)\}, \tag{3.2}$$

where $H^m(B)$ are the Sobolev space and $\boldsymbol{H}^m(B) = [H^m(B)]^3$. Consider now the following linear operators on X

$$\begin{cases} A_i w = v_i, \quad B_i w = (C_{ijrs}u_{r,sj} - D_{kij}e_{k,j})/\varrho, \\ C_i w = \chi_{ij}(c\varepsilon_{jrs}b_{s,r} - D_{jrs}v_{r,s}), \quad D_i w = -c\varepsilon_{irs}e_{s,r}. \end{cases} \tag{3.3}$$

Let A be the operator

$$Aw = (A_i w, B_i w, C_i w, D_i w), \tag{3.4}$$

with the domain

$$D(A) = \{w = (\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{e}, \boldsymbol{b}) \in X; Aw \in X, \varepsilon_{ijk}e_j n_k = 0 \text{ on } \overline{S}_1, \varepsilon_{ijk}b_j n_k = 0 \text{ on } S_2\}. \tag{3.5}$$

Clearly, $D(A)$ is dense in X. The boundary-initial-value problems (2.9), (2.11), (2.12) can be transformed into the following equation in the Hilbert space X

$$\begin{cases} dw(t)/dt = Aw(t) + F(t)\,, & t > 0\,, \\ w(0) = w_0\,, \end{cases} \tag{3.6}$$

where

$$F = (\mathbf{0}, f, \mathbf{0}, \mathbf{0})\,, \qquad w_0 = (\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}, \hat{\boldsymbol{e}}, \hat{\boldsymbol{b}})\,.$$

Let X_* be the Hilbert space X equipped with the norm $\|\cdot\|_*$ induced by the inner product

$$\langle w, \overline{w}\rangle_* = \int_B (C_{ijrs} u_{r,s} \overline{u}_{i,j} + \varrho v_i \overline{v}_i + \Gamma_{ij} e_i \overline{e}_j + b_i \overline{b}_i)\, dv\,. \tag{3.7}$$

By the hypotheses (3.1) and the first Korn inequality we conclude that the norm $\|\cdot\|_*$ is equivalent to the original norm $\|\cdot\|$ in X.

Lemma 3.1. The operator A is dissipative.

Proof. By (3.3), (3.4) and (3.7),

$\langle Aw, w\rangle_* =$

$$= \int_B [C_{ijrs} u_{r,s} v_{i,j} + v_i (C_{ijrs} u_{r,sj} - D_{kij} e_{k,j}) + \Gamma_{ij} e_i \chi_{jk} (c\varepsilon_{krs} b_{s,r} - D_{krs} v_{r,s}) - c b_i \varepsilon_{irs} e_{s,r}]\, dv\,.$$

Using (2.10) and the divergence theorem we obtain

$$\langle Aw, w\rangle_* = \int_{\partial B} [v_i (C_{ijrs} u_{r,s} - D_{kij} e_k) n_j - c\varepsilon_{irs} e_s b_i n_r]\, da\,.$$

The boundary conditions (2.12) imply

$$\langle Aw, w\rangle_* \leqslant 0 \qquad \text{for every } w \in D(A)\,.$$

The proof is complete.

We now consider the operator $\lambda I - A$ where I is the identity operator and $\lambda > 0$.

Lemma 3.2. The operator A satisfies the range condition

$$R(\lambda I - A) = X\,, \qquad \lambda > 0\,.$$

Proof. Let $\overline{w} = (\overline{\boldsymbol{u}}, \overline{\boldsymbol{v}}, \overline{\boldsymbol{e}}, \overline{\boldsymbol{b}}) \in X$. We must prove that the equation

$$\lambda w - Aw = \overline{w}\,, \qquad \lambda > 0\,, \tag{3.8}$$

has a solution $w = (\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{e}, \boldsymbol{b})$ in $D(A)$. By eliminating $\boldsymbol{v}$, (3.8) yields the following system for $\boldsymbol{u}$, $\boldsymbol{e}$ and $\boldsymbol{b}$

$$\begin{cases} L_i y \equiv \lambda^2 u_i - (C_{ijrs} u_{r,sj} - D_{kij} e_{k,j})/\varrho = g_i\,, \\ M_i y \equiv \lambda e_i - \chi_{ij} (c\varepsilon_{jrs} b_{s,r} - \lambda D_{jrs} u_{r,s}) = h_i\,, \\ N_i y \equiv \lambda b_i + c\varepsilon_{irs} e_{s,r} = \overline{b}_i\,, \end{cases} \tag{3.9}$$

where

$$y = (u_i, e_i, b_i), \qquad g_i = \bar{v}_i + \lambda \bar{u}_i, \qquad h_i = \bar{e}_i - \chi_{ij} D_{jrs} \bar{u}_{r,s}. \tag{3.10}$$

Let $[\cdot, \cdot]$ denote a conveniently weighted $L_2(B) \times L_2(B) \times L_2(B)$ inner product, and consider the bilinear form

$$G(y, \tilde{y}) = [(L_i y, M_i y, N_i y), (\tilde{u}_i, \tilde{e}_i, \tilde{b}_i)] = \int_B (\varrho \lambda \tilde{u}_i L_i y + \Gamma_{ij} \tilde{e}_i M_j y + \tilde{b}_i N_i y)\, dv. \tag{3.11}$$

The divergence theorem and the boundary conditions imply

$$G(y, y) = \int_B \lambda(\lambda^2 \varrho u_i u_i + C_{ijrs} u_{r,s} u_{i,j} + \Gamma_{ij} e_i e_j + b_i b_i)\, dv + \int_{\partial B} (\lambda \nu^{-1} u_i u_i + \delta^{-1} e_i e_i)\, d\sigma \tag{3.12}$$

for any $y = (\boldsymbol{u}, \boldsymbol{e}, \boldsymbol{b}) \in Y \equiv \boldsymbol{H}_0^1(B) \times \boldsymbol{H}^0(B) \times \boldsymbol{H}^0(B)$. By (3.1), (3.12) and the first Korn inequality [12],

$$G(y, y) \geq a \|y\|_Y^2 \quad \text{for every } y \in Y, \tag{3.13}$$

where $a = \min(\lambda^2 c_1, \lambda c_0 c_1, \lambda \varepsilon_0, \lambda)$, $\|y\|_Y^2 = \|(\boldsymbol{u}, \boldsymbol{e}, \boldsymbol{b})\|_Y^2 = \|\boldsymbol{u}\|_{H^1(B)}^2 + \|\boldsymbol{e}\|_{H^0(B)}^2 + \|\boldsymbol{b}\|_{H^0(B)}^2 + \|\boldsymbol{u}\|_{H^0(\partial B)}^2 + \|\boldsymbol{e}\|_{H^0(\partial B)}^2$, and c_1 is the constant from the first Korn inequality.

Since the bilinear form $G(y, \tilde{y})$ is continuous in $Y \times Y$, there exists a linear bounded transformation T from Y into itself such that

$$G(y, \tilde{y}) = [y, T\tilde{y}]_Y, \tag{3.14}$$

for any $y, \tilde{y} \in Y$. Since $|[y, Ty]_Y| \geq a\|y\|_Y^2$, we have $\|Ty\|_Y \geq a\|y\|_Y$, for every $y \in Y$.

Let $R(T)$ be the range of T. Let $y_0 \in Y$ such that $Ty_0 = 0$. By (3.14) we obtain $G(y_0, y_0) = 0$ and (3.13) implies $y_0 = 0$. Thus, we conclude that T is one to one. Therefore, there exists $T^{-1}: R(T) \to Y$. We can also prove that $R(T)$ is dense in Y. Then, we can continue T^{-1} to Y. For any $z \in R(T)$, set $\varphi(z) = [(g_i, h_i, \bar{b}_i), \omega]_Y$ where ω is the only element of Y such that $z = T\omega$. Then, φ is a linear bounded functional defined on $R(T)$. We can continue φ in the whole space Y, in such a way that the continued functional Φ shall have the same norm. Since Y is a Hilbert space, there exists a unique $y^* \in Y$ such that

$$\Phi(y) = [y^*, y]_Y, \qquad \text{for any } y \in Y. \tag{3.15}$$

If we choose $y = T\tilde{y}$, then (3.14) and (3.15) imply that $y^* = (u_i^*, e_i^*, b_i^*) \in Y$ satisfies the equation

$$G(y^*, \tilde{y}) = [(g_i, h_i, \bar{b}_i), \tilde{y}]_Y \qquad \text{for every } \tilde{y} \in Y.$$

Thus, $L_i y^* = g_i$, $M_i y^* = h_i$, $N_i y^* = \bar{b}_i$. It follows from $v_i^* = \lambda u_i^* - \bar{u}_i$ that $\boldsymbol{v}^* \in \boldsymbol{H}^0(B)$. We conclude that $(\boldsymbol{u}^*, \boldsymbol{v}^*, \boldsymbol{e}^*, \boldsymbol{b}^*) \in D(A)$. ■

THEOREM 3.1. The operator A generates a C_0 semigroup of contractions on X.

PROOF. The proof follows from the Lemmas 3.1, 3.2 and the Lumer-Phillips theorem (see, *e.g.*, [13, p. 13]).

We now state the following result (see, for example, Pazy [13, Chapter 4]).

THEOREM 3.2. Let A be the infinitesimal generator of a C_0 contractive semigroup. If F is continuously differentiable on $[0, t_1]$ then the initial value problem (3.6) has, for every $w_0 \in D(A)$, a unique solution $w \in C^1([0, t_1]; X) \cap C^0([0, t_1]; D(A))$.

The next theorem is an immediate consequence of Theorems 3.1 and 3.2.

THEOREM 3.3. Assume that the density field is strictly positive and the constitutive coefficients satisfy the conditions (2.6) and (3.1). Further, assume that $f \in C^1([0, t_1]; L_2(B))$ and $w_0 = (\widehat{\boldsymbol{u}}, \widehat{\boldsymbol{v}}, \widehat{\boldsymbol{e}}, \widehat{\boldsymbol{b}}) \in D(A)$. Then there exists a unique solution $w \in C^1([0, t_1]; X) \cap C^0([0, t_1]; D(A))$ to be boundary-initial-value problem (2.9), (2.11), (2.12).

Now we establish the continuous dependence of the solution upon the initial data and body forces.

THEOREM 3.4. Assume that the density field is strictly positive and that (3.1) holds. Further, assume that $f \in L_1([0, t_1]; L_2(B))$ and $\widehat{\boldsymbol{u}} \in \boldsymbol{H}_0^1(B)$, $\widehat{\boldsymbol{v}} \in \boldsymbol{H}^0(B)$, $\widehat{\boldsymbol{e}} \in \boldsymbol{H}^0(B)$, $\widehat{\boldsymbol{b}} \in \boldsymbol{H}^0(B)$.

Let $(\boldsymbol{u}, \boldsymbol{e}, \boldsymbol{b})$ be the solution of the boundary-initial-value problem (2.9), (2.11), (2.12) corresponding to the body force f and the initial data $(\widehat{\boldsymbol{u}}, \widehat{\boldsymbol{v}}, \widehat{\boldsymbol{e}}, \widehat{\boldsymbol{b}})$. Let M be the positive function on $[0, t_1]$ defined by

$$M^2 = \|\boldsymbol{u}\|^2_{H^1(B)} + \|\dot{\boldsymbol{u}}\|^2_{H^0(B)} + +\|\boldsymbol{e}\|^2_{H^0(B)} + \|\boldsymbol{b}\|^2_{H^0(B)} \,.$$

Then there exists a positive constant α such that

$$M(t) \leqslant \alpha\left[M(0) + \int_0^t \|\varrho f\|_{H^0(B)}\, d\tau\right], \qquad t \in [0, t_1]. \tag{3.16}$$

PROOF. By (2.5) and (2.6),

$$t_{ij}\dot{E}_{ij} + e_i\dot{d}_i + b_i\dot{b}_i = \frac{1}{2}\frac{\partial}{\partial t}(C_{ijrs}E_{ij}E_{rs} + \Gamma_{ij}e_ie_j + b_ib_i)\,. \tag{3.17}$$

On the other hand, from (2.1), (2.3), (2.4) and (2.6) we obtain

$$\begin{aligned} t_{ij}\dot{E}_{ij} + e_i\dot{d}_i + b_i\dot{b}_i &= (t_{ij}\dot{u}_i)_{,j} - t_{ji,j}\dot{u}_i + c\varepsilon_{irs}b_{s,r}e_i + c\varepsilon_{irs}b_se_{i,r} = \\ &= \varrho f_i\dot{u}_i - \varrho\ddot{u}_i\dot{u}_i + (t_{ij}\dot{u}_i)_{,j} + (c\varepsilon_{irs}b_se_i)_{,r}\,. \end{aligned}$$

By the divergence theorem and (2.12),

$$\int_B (t_{ij}\dot{E}_{ij} + e_i\dot{d}_i + b_i\dot{b}_i)\, dv = \int_B \varrho(f_i - \ddot{u}_i)\,\dot{u}_i\, dv\,. \tag{3.18}$$

Let E be the function on $[0, t_1]$ defined by

$$E = \int_B (\varrho \dot{u}_i \dot{u}_i + C_{ijrs} E_{ij} E_{rs} + \Gamma_{ij} e_i e_j + b_i b_i)\, dv\,. \tag{3.19}$$

It follows from (3.17) and (3.18) that

$$\dot{E} \leq 2 \int_B \varrho f_i \dot{u}_i\, dv\,. \tag{3.20}$$

Then, we have

$$E(t) \leq E(0) + 2 \int_0^t \int_B \varrho f_i \dot{u}_i\, d\tau\, dv\,, \qquad t \in [0, t_1]\,. \tag{3.21}$$

By the Schwarz inequality,

$$E(t) \leq E(0) + 2 \int_0^t \|\varrho \boldsymbol{f}\|_{H^0(B)} \|\dot{\boldsymbol{u}}\|_{H^0(B)}\, d\tau\,. \tag{3.22}$$

By using the first Korn inequality, (3.1) and the assumption that ϱ be strictly positive, we can determine a positive constant m_0 such that

$$M^2(t) \leq m_0 E(t)\,, \qquad t \in [0, t_1]\,. \tag{3.23}$$

On the other hand, we can determine a positive constant m_1 such that

$$E(0) \leq m_1 M^2(0)\,. \tag{3.24}$$

It follows from (3.22), (3.23) and (3.24) that

$$M^2(t) \leq m_0 m_1 M^2(0) + 2m_0 \int_0^t \|\varrho \boldsymbol{f}\|_{H^0(B)} M\, d\tau\,, \qquad t \in [0, t_1]\,.$$

This is a Gronwall-type inequality, so that [14]

$$M(t) \leq \sqrt{m_0 m_1}\, M(0) + m_0 \int_0^t \|\varrho \boldsymbol{f}\|_{H^0(B)}\, d\tau\,, \qquad t \in [0, t_1]\,. \tag{3.25}$$

The desired result is an immediate consequence of (3.25). ■

Some asymptotic and Liapounov stability results have been established in [15].

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