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# HUA-HARMONIC FUNCTIONS ON SYMMETRIC TYPE TWO SIEGEL DOMAINS

ABSTRACT. — We study a natural system of second order differential operators on a symmetric Siegel domain $\mathcal{D}$ that is invariant under the action of biholomorphic transformations. If $\mathcal{D}$ is of type two, the space of real valued solutions coincides with pluriharmonic functions. We show the main idea of the proof and give a survey of previous results.

KEY WORDS: Symmetric Siegel domain; Pluriharmonic function; Invariant system of differential operators.

## 1. SIEGEL DOMAINS

Symmetric Siegel domains are in 1-1 correspondence with bounded symmetric domains in $\mathbb{C}^n$ via biholomorphic mappings [15]. We study a natural $G$-invariant system of second order operators defined equivalently in both realization. However, for the technics we use, the unbounded realization is more convenient.

A Siegel tube type domain is the domain

$$\mathcal{D} = V + i\Omega \subset V^{\mathbb{C}} ,$$

where $\Omega$ is a symmetric cone in a Euclidean space $V$. The most natural example is $V$ being the space of $r \times r$ real symmetric matrices and $\Omega$ the cone of positive definite matrices in $V$.

Suppose that, we are given a complex vector space $\mathcal{Z}$ and a Hermitian bilinear mapping

$$\Phi : \ \mathcal{Z} \times \mathcal{Z} \to V^{\mathbb{C}} .$$

We assume that

$$\Phi(\zeta , \zeta) \in \overline{\Omega} , \quad \zeta \in \mathcal{Z} ,$$
$$\text{and } \Phi(\zeta , \zeta) = 0 \ \text{ implies } \zeta = 0 .$$

The Siegel domain of type two associated with these data is defined as

$$\mathcal{D} = \left\{ (\zeta , z) \in \mathcal{Z} \times V^{\mathbb{C}} : \ \Im z - \Phi(\zeta , \zeta) \in \Omega \right\} .$$

The simplest example of such a domain is

$$\mathcal{D}_m = \left\{ (\zeta , z) \in \mathbb{C}^m \times \mathbb{C} : \Im z - |\zeta|^2 > 0 \right\} ,$$

which is biholomorphically equivalent to the unit ball

$$\left\{ w \in \mathbb{C}^{m+1} : |w|^2 < 1 \right\} .$$

Let $G(\mathcal{D})$ be the connected component of the group of biholomorphic transformations of $\mathcal{D}$ and let $S$ be the solvable part of its Iwasawa decomposition. $S$ acts simply transitively on $\mathcal{D}$ and it will be identified with it.

## 2. Hua-harmonic functions

To define the Hua system we need some notation. Let $T$ be the tangent bundle for $\mathcal{D}$ and $T^{\mathbb{C}}$-the complexified tangent bundle. We write $T^{\mathbb{C}}$ as

$$T^{\mathbb{C}} = T^{1,0} \oplus T^{0,1} ,$$

with $T^{1,0}$ being the holomorphic tangent bundle and $T^{0,1}$-the antiholomorphic tangent bundle. We choose an orthonormal frame $Z_1, \dots, Z_m$ in $T^{1,0}$ and we write

$$\mathbf{HJK}(F) = \sum_{j,k} \left( (Z_j \bar{Z}_k - \nabla_{Z_j} \bar{Z}_k) F \right) \ R(\bar{Z}_j, Z_k)|_{T^{1,0}} ,$$

$\nabla$ is the Bergman connection and $R$-the curvature tensor. $Z_j \bar{Z}_k - \nabla_{Z_j} \bar{Z}_k$ is the unique $S$-invariant operator corresponding to $\partial_{z_j} \partial_{\bar{z}_k}$ at a given point.

Given $F$, $\mathbf{HJK}(F)$ is a section of endomorphisms of $T^{1,0}(\mathcal{D})$ invariant in the following sense:

$$\mathbf{HJK}(F \circ \Psi) = \Psi_*^{-1} \circ \mathbf{HJK}(F) \circ \Psi_* ,$$

for every biholomorphic transformation $\Psi$ of $\mathcal{D}$.

The system **HJK** can be written on any Kählerian manifold, it is invariant and it annihilates holomorphic and antiholomorphic functions. Moreover, for tube domains it coincides with the system written by A. Korányi, E. Stein and J. Wolf in the sixties. It took more then twenty years to characterize zeros of **HJK** for the tube case. The story started in 1958 when L.H. Hua [9] wrote the system for some classical domains and he proved that it annihilates the Poisson-Szegö kernel. Then A. Korányi, E. Stein and J. Wolf obtained the formula for general tube domains and in an unpublished paper showed that the Poisson-Szegö kernel is harmonic with respect to the system (see *e.g.* [12]). The first results showing that differential equations actually characterize the class of Poisson-Szegö integrals were obtained in special cases [13, 10, 11]. Finally in 1980 K. Johnson and A. Korányi proved the following theorem:

THEOREM 1 (K. Johnson and A. Korányi, 1980). *A function $F$ on a symmetric tube domain satisfies* $\mathbf{HJK}(F) = 0$ *if and only if it is the Poisson-Szegö integral of a hyperfunction.*

The Johnson and Korányi theorem [12] shows that in the tube case the system is closely related to the Shilov boundary. The question what is the meaning of the system for non-tube type symmetric Siegel domains remained open for next twenty years. It was explicitly formulated in the paper by N. Berline and M. Vergne [1], where they described a third order system that characterizes Poisson-Szegö integrals on type two symmetric Siegel domains.

It turns out that the Hua-Johnson-Korányi system on type two Siegel domains characterizes pluriharmonicity [3]:

THEOREM 2 (D. Buraczewski, 2001). *Let $F$ be a real valued function on a non-tube irreducible symmetric domain. If* $\mathbf{HJK}(F) = 0$ *then $F$ is pluriharmonic.*

Theorem 2 says that although the **HJK** system is defined by such a natural geometric objects like the connection and the curvature tensor, in general, it is not much hope to obtain anything more than pluriharmonic functions as its zeros.

The proof of the above theorem consists of two steps: first, to prove it for bounded functions and secondly, to write an arbitrary Hua-harmonic function as a series of $K$-finite bounded Hua-harmonic functions. Of course, the most important job is done within the first step and it relies heavily on the identification of $\mathcal{D}$ with the solvable group $S$.

## 3. BOUNDED HJK-HARMONIC FUNCTIONS

In this section we are going to sketch the proof of Theorem 2 for bounded functions. Before that we need some biographical comments. The idea that **HJK**-harmonic functions on Siegel type two domains could possibly be pluriharmonic appeared as a result of our previous studies of pluriharmonicity there [4-6]. In these papers we characterized pluriharmonicity within the class of bounded functions on type two Siegel domains by means of at most three elliptic degenerate operators. Moreover, we described a large class of operators doing the job. It was quite natural to expect that the methods should be applicable to the Hua system. The first result in this direction was:

THEOREM 3 (A. Bonami, D. Buraczewski, E. Damek, A. Hulanicki, R. Penney and B. Trojan, 1999). *Let $F$ be a real valued function satisfying the following condition*

$$\sup_{s\in S}\int_{N(\Phi)} |F(us)|^2\, du \langle \infty$$

*on a non-tube irreducible symmetric domain. If* $\mathbf{HJK}(F) = 0$ *then $F$ is the real part of a holomorphic $H^2$ function.*

In all our studies of pluriharmonicity, the crucial point was to use the solvable group $S$. This way we focused our attention on objects invariant under the group $S$ rather then $G(\mathcal{D})$-invariant ones. Notice that there are plenty of functions which are not pluriharmonic and are annihilated by all the $G(\mathcal{D})$-invariant operators without a constant term. Therefore, a more traditional approach to analysis on symmetric spaces misses pluriharmonicity.

Both characterizing pluriharmonic functions and describing zeros of the **HJK**-system we started with the above $H^2$ condition, because functions in this space were easier to handle via the Fourier transform. When methods of treating bounded functions in this context have been elaborated [4] we generalized our theorems.

We study $S$-invariant operators that arise from the **HJK** system and map functions into functions. Given $Z, W \in T^{1,0}$ let

$$\mathbf{HJK}_{Z,W}F = \langle \mathbf{HJK}(F)(Z)\,,\, W\rangle\,.$$

This way we obtain a number of second order operators. To write them more explicitly we have to understand better the structure of the group $S$. Its Lie algebra is a semi-direct sum:

$$\mathcal{S} = \mathcal{N} \oplus \mathcal{A}\,, \qquad \mathcal{N} = \oplus_{\eta\in\Delta}\mathcal{N}_\eta\ ,$$

with $\mathcal{A}$ acting diagonally on $\mathcal{N}$:

$$[H, X] = \eta(H)X, \qquad X \in \mathcal{N}_\eta\,, \quad \eta \in \Delta \subset \mathcal{A}^*\,.$$

The corresponding groups will be denoted by:

$$N = \exp\mathcal{N}\,, \qquad A = \exp\mathcal{A}\,, \qquad S = \exp\mathcal{S} = NA\,.$$

The action of $A$ on $N$ is diagonal with eigenvalues $e^{\eta(\log a)}$:

$$a \exp Xa^{-1} = \exp e^{ad_{\log\ a}}X\,.$$

The set of roots $\Delta$ has the following structure:

$$\Delta = \{\lambda_1\,, \dots\,, \lambda_r\} \cup \Delta'\ ,$$

where $\Delta'$ consists of linear combinations of the roots $\lambda_1\,, \dots\,, \lambda_r$. For symmetric tube domains we have:

$$\Delta' = \left\{\frac{\lambda_i+\lambda_j}{2}\,, \frac{\lambda_j-\lambda_i}{2}\,,\ 1\le i<j\le r\right\}$$

$$\dim\mathcal{N}_{\lambda_j} = 1$$

$$\dim\mathcal{N}_{\frac{\lambda_j+\lambda_i}{2}} = \dim\mathcal{N}_{\frac{\lambda_j-\lambda_i}{2}} = d,\ 1\le i<j\le r\ ,$$

while for type two symmetric domains:

$$\Delta' = \left\{\frac{\lambda_i+\lambda_j}{2}\,, \frac{\lambda_j-\lambda_i}{2}\,,\ 1\le i<j\le r, \quad \frac{\lambda_j}{2}\,,\ 1\le j\le r\right\}\ ,$$

$$\dim\mathcal{N}_{\lambda_j} = 1\ ,$$

$$\dim\mathcal{N}_{\frac{\lambda_j+\lambda_i}{2}} = \dim\mathcal{N}_{\frac{\lambda_j-\lambda_i}{2}} = d,\ 1\le i<j\le r\ ,$$

$$\dim\mathcal{N}_{\frac{\lambda_j}{2}} = \chi\,,\ 1\le j\le r\,.$$

The Lie algebra $\mathcal{S}$ decomposes as

$$\mathcal{S} = \mathcal{Z} \oplus V \oplus \mathcal{N}_0 \oplus \mathcal{A}\ ,$$

where

$$\mathcal{Z} = \oplus_{j=1}^r\mathcal{N}_{\frac{\lambda_j}{2}}\ ,$$

$$V = \oplus_{i\le j}\mathcal{N}_{\frac{\lambda_i+\lambda_j}{2}}\ ,$$

$$\mathcal{N}_0 = \oplus_{i<j}\mathcal{N}_{\frac{\lambda_j-\lambda_i}{2}}\ .$$

Let

$$N(\Phi) = \exp(\mathcal{Z} \oplus V)\,, \qquad N_0 = \exp\mathcal{N}_0\,, \qquad V = \exp V\,.$$

Then

$$S = N(\Phi)N_0A\ ,$$

in the sense that any $s \in S$ can be written as

$$s = wya = (\zeta\,,\,x)ya\ ,$$
$$w = (\zeta\,,\,x) \in N(\Phi)\,,\, y \in N_0\,,\, a \in A\,.$$

In these terms $S = VN_0A$ is a solvable group acting simply transitively on the tube domain

$$\mathcal{D}_T = V + i\Omega = \{(0\,,\,z) : \Im z \in \Omega\} \subset \mathcal{D}\,.$$

Clearly, for its Lie algebra $\mathcal{S}_T$ we have:

$$\mathcal{S}_T = V \oplus \mathcal{N}_0 \oplus \mathcal{A}\,.$$

Let $F$ be a function on $\mathcal{D}$. Then

$$F_\zeta(xya) = F\big((\zeta\,,\,x)ya\big) = F\big((\zeta\,,\,0)(0\,,\,x)ya\big)$$

is a function on $\mathcal{D}_T$ and left-invariant operators on $VN_0A$ (*i.e.* $VN_0A$-invariant operators on $V + i\Omega$) are well defined when applied to $F$. We are going to make use of that.

Let $Z,\,W$ be $S$-invariant sections of $T^{1,0}$. Then the operators

$$\mathbf{HJK}_{Z,W}F = \langle \mathbf{HJK}(F)(Z)\,,\,W\rangle$$

are left-invariant on $S$. Clearly,

$$\mathbf{HJK}(F) = 0 \iff \mathbf{HJK}_{Z,W}(F) = 0\ ,$$

for all $S$-invariant $Z,\,W \in T^{1,0}$.

The first step is to prove that a Hua-harmonic function is the Poisson-Szegö integral:

$$F\big((\zeta\,,\,x)ya\big) = \int_{N(\Phi)} f\big((\zeta\,,\,x)yawa^{-1}y^{-1}\big)\,P(w)\,dw\ , \tag{1}$$

$P$ being the Poisson-Szegö kernel for $\mathcal{D}$. A simple proof of that can be found in [2].

Next, we prove that the Laplace-Beltrami operator $\Delta_T$ for $\mathcal{D}_T$ is among the operators $\mathbf{HJK}_{Z,W}$. This implies that for every $\zeta$,

$$F_\zeta(xya) = \int_{VN_0} f_\zeta(xyavna^{-1})p(vn)\,dvdn\ , \tag{2}$$

where $p$ is the Poisson kernel for $\mathcal{D}_T$. Letting $a \to 0$ in both (1) and (2) we get

$$f(\zeta\,,\,x) = f_\zeta(xy)\,.$$

Therefore,

$$f_\zeta(xyavna^{-1}) = f_\zeta\big(xyav(ya)^{-1}\big)$$

and by (2)

$$F\big((\zeta\,,\,x)ya\big) = \int_V f_\zeta\big(xyav(ya)^{-1}\big)\,q(v)\,dv\ ,$$

where

$$q(v) = \int_{N_0} p(vn)\,dn$$

is the Poisson-Szegö kernel for the tube $V + i\Omega$. Now applying the Johnson, Korányi result (Theorem 1), we see that

$$\mathbf{HJK}_T(F) = 0$$

and we obtain new linearly independent equations [2].

## 4. Induction and reduction to the complex ball

The rest of the proof goes by induction on the rank $r$ of the cone and the main work is done on the complex ball. For that we have to decompose the group $S$ properly. Let

$$\mathcal{N}_{ij} = \mathcal{N}_{\frac{\lambda_j - \lambda_i}{2}}\,, \qquad V_{ij} = \mathcal{N}_{\frac{\lambda_i + \lambda_j}{2}}\,, \qquad \mathcal{Z}_j = \mathcal{Z}_{\frac{\lambda_j}{2}}$$

and let

$$\mathcal{H}_r = \mathcal{Z}_r \oplus_{j<r} (V_{jr} \oplus \mathcal{N}_{jr}) \oplus V_{rr}\,.$$

Then it can be easily seen that $\mathcal{H}_r$ is the Heisenberg with the centre $V_{rr}$. Let $A_r = \exp \mathbb{R} H_r$, where $H_r$ is the dual vector to $\lambda_r$ and let $S_r = \exp \mathcal{H}_r A_r$. Then $S_r$ is the group acting simply transitively on the Siegel half plane $\mathcal{D}_r$ which is an unbounded realization of the complex ball. $S_r$ will be identified with $\mathcal{D}_r$. The crucial observation is that $S$ is a semi-direct product

$$S = S' S_r\,,$$

where $S'$ a group acting simply transitively on the Siegel domain $\mathcal{D}'$ of the rank $r-1$. $S_r$ is normal in $S$.

It turns out that some **HJK** operators are operators on $S_r$. So we can restrict our function to left cosets of $S_r$ and work there. Let

$$\begin{aligned}
\mathcal{X}_r^\alpha\,, \mathcal{Y}_r^\alpha\,, \alpha = 1\,, \dots\,, \chi &\text{ be a basis of } \mathcal{Z}_r\\
X_{jr}^\alpha\,, \alpha = 1\,, \dots\,, d &\text{ be a basis of } V_{jr}\\
Y_{jr}^\alpha\,, \alpha = 1\,, \dots\,, d &\text{ be a basis of } \mathcal{N}_{jr}\\
X_{rr} \qquad\qquad &\text{ be a basis of } V_{rr}
\end{aligned}$$

orthonormal in Bergman metric for $\mathcal{D}$. Then it is orthonormal in the Bergman metric for $\mathcal{D}_r$ as well. Moreover, the complex structure $J$ coincides on both $\mathcal{D}$ and $\mathcal{D}_r$ and

$$\begin{aligned}
J(\mathcal{X}_r^\alpha) &= \mathcal{Y}_r^\alpha\\
J(X_{jr}^\alpha) &= Y_{jr}^\alpha\\
J(X_{rr}) &= H_r\,,
\end{aligned}$$

[3]. Among **HJK**'s are the following operators:

$$\mathcal{L} = \sum_{\alpha=1}^{\chi} (\mathcal{X}_r^\alpha)^2 + (\mathcal{Y}_r^\alpha)^2 - \chi H_r \,,$$

$$L = \frac{1}{2} \sum_{\substack{j=1 \\ \alpha}}^{r-1} (X_{jr}^\alpha)^2 + (Y_{jr}^\alpha)^2 + X_{rr}^2 + H_r^2 - \left( \frac{1}{2}(r-1)d + 1 \right) H_r$$

(see [3]). In this notation the sublaplacian on $\exp \mathcal{H}_r$ is

$$L_B f(w) = \sum_{\alpha=1}^{\chi} (\mathcal{X}_r^\alpha)^2 + (\mathcal{Y}_r^\alpha)^2 + \sum_{\substack{j=1 \\ \alpha}}^{r-1} (X_{jr}^\alpha)^2 + (Y_{jr}^\alpha)^2|_{a=1} f(w) \,.$$

(An element $s$ of $S_r$ is written as $s = wa$, $w \in \exp \mathcal{H}_r$, $a \in A_r$). The fact that $\mathcal{L}$ annihilates $F|_{S_r}$ allows us to eliminate

$$H_r = \frac{1}{\chi} \sum_{\alpha=1}^{\chi} (\mathcal{X}_r^\alpha)^2 + (\mathcal{Y}_r^\alpha)^2$$

and we get the boundary equation

$$(L_B^2 + m^2 T^2) f_r = 0 \,, \tag{3}$$

which implies that $F|_{S_r}$ is pluriharmonic [4]. In (3) $f_r$ is the boundary value of $F|_{S_r}$ on $\exp \mathcal{H}_r$ and $T = X_{rr}|_{a=1}$.

Pluriharmonicity of $F|_{S_r}$ (as well as of $F$ restricted to left cosets of $S_r$) implies that $F$ satisfies extra equations. Using them we are able to prove that $F|_{S'}$ and all its left translates are annihilated by the **HJK**-system for $\mathcal{D}'$ and we may proceed by induction [3].

## 5. HJK-HARMONIC FUNCTIONS

To treat arbitrary **HJK**-harmonic functions we have to prove that **HJK**$(F) = 0$ implies that $F$ is $G(\mathcal{D})$-harmonic. Then we can write

$$F = \sum_{\pi \in \hat{K}} F_\pi \,,$$

where $F_\pi = \chi_\pi *_K F$ is the projection of $F$ onto the space of $K$-finite vectors of type $\pi$ [7]. $\hat{K}$ is the set of equivalence classes of irreducible unitary representations of $K$ and $\chi_\pi$ is the character of $\pi$. Each $F_\pi$ is clearly Hua harmonic and so $G(\mathcal{D})$ harmonic. But a $K$-finite $G(\mathcal{D})$ harmonic function is bounded [8]. Hence Theorem 2 for bounded functions sais that every $F_\pi$ is pluriharmonic and so is $F$.

There are two approaches to prove strong harmonicity of $F$. The first one is due to Johnson and Korányi [12] the second one – to Lassalle [14]. In the latter one the following lemma is crucial:

LEMMA 1 (M. Lassalle – tube type, D. Buraczewski – non tube). *Let $F$ be a bi-$K$-invariant function on the semi-simple Lie group $G(\mathcal{D})$ and* $\mathbf{HJK}(F) = 0$ *then $F$ is constant.*

In the above lemma both the system and the function are lifted to $G(\mathcal{D})$ (see [3, 14]). While Lemma 1 is proved, for a **HJK**-harmonic function $F$ we write

$$\Phi(x) = \int_K F(gkx)\ dk\,, \quad x \in \mathcal{D}$$

$\Phi$ lifted to $G(\mathcal{D})$ is bi-$K$-invariant and, clearly, annihilated by the Hua system:

$$\mathbf{HJK}(\Phi)(x) = \int_K \mathbf{HJK}(F)(gkx)\ dk = 0\,.$$

Therefore $\Phi$ is constant and so

$$\int_K F(gkx)\ dk = \Phi(x) = \Phi(x_0) = F(gx_0)\ ,$$

which means that $F$ is $G(\mathcal{D})$-harmonic.

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