Matematica, Cultura e Società

Rivista dell'Unione Matematica Italiana

Nicla Palladino

Giovanni Novi's contributions to theteaching of Geometry

Matematica, Cultura e Società. Rivista dell'Unione Matematica Italiana, Serie 1, Vol. **6** (2021), n.2, p. 147–160.

Unione Matematica Italiana

<http://www.bdim.eu/item?id=RUMI_2021_1_6_2_147_0>

Matematica, Cultura e Società. Rivista dell'Unione Matematica Italiana, Unione Matematica Italiana, 2021.

Matematica, Cultura e Società – Rivista dell'Unione Matematica Italiana
Serie I, Vol. 6, N. 2, agosto 2021, 147-160

Giovanni Novi's contributions to the teaching of Geometry

Nicla Palladino
Università di Perugia
E-mail: nicla.palladino@unipg.it

Sommario: *A seguito della scoperta e dello studio delle lettere intercorse tra i matematici italiani Enrico Betti e Giovanni Novi, è stato possibile ricostruire l'attività scientifica di Novi, finora non molto nota. Egli contribuì al rinnovamento dell'insegnamento della geometria nel periodo immediatamente precedente all'Unità d'Italia, soprattutto con un lavoro di traduzione ed integrazione di trattati matematici stranieri. In particolare, nel 1858 Novi pubblicò la traduzione del testo francese di Antoine Amiot* Leçons nouvelles de géométrie élémentaire. *La traduzione fu molto apprezzata da illustri matematici italiani poiché arricchì il trattato di contributi originali sulla geometria proiettiva. Dopo un'analisi del testo italiano, possiamo concludere che questo può essere considerato un'anticipazione di quelli che saranno i più diffusi manuali italiani nel periodo successivo, prefiggendosi lo scopo di integrare l'insegnamento della geometria con la ricerca scientifica sulle relazioni tra la matematica pura e le sue applicazioni. Qui contestualizziamo questa operazione nel periodo storico in cui Novi e altri illustri matematici lavorarono.*

Abstract: *Thanks to the discovery and analysis of letters exchanged between the Italian mathematicians Enrico Betti and Giovanni Novi, we have been able to reconstruct the scientific activity of Novi, that until recently was not well known. Novi contributed to renovating the teaching of geometry in the period immediately preceding the Italian Unification by translating and integrating foreign mathematical treatises. In particular, in 1858, he published a translation of the French textbook by Antoine Amiot* Leçons nouvelles de géométrie élémentaire. *This translation was greatly appreciated because Novi integrated the treatise with original contributions on projective geometry. After careful analysis of his text, we conclude that it can be considered an anticipation of what will become the most popular Italian texts aiming at connecting the teaching of geometry with the scientific research on the relations between pure mathematics and its applications. We contextualize this operation in the historical context in which famous Italian mathematicians worked.*

Introduction

Studying the 48 letters that the Neapolitan mathematician Giovanni Novi wrote to Enrico Betti between 1850 and 1864 allows us to reconstruct Novi's scientific activity [Mercurio-Palladino 2011]. The correspondence, stored at the Archivio Betti – Biblioteca of the Scuola Normale Superiore of Pisa, is an essential document outlining the figure of the mathematician and his works that are still not well known, but already evident from other correspondences and documents, such as the letters between Cesira Pozzolini and Pietro Siciliani ([Luceri 2013], [Luceri 2015], [Pozzolini 1875], [Pozzolini 1904]), "Il carteggio Betti-Tardy (1850-1891)" [Cerroni-Martini 2010], the registers of the Accademia della Nunziatella, the Office of topography and State archive

Accettato: il 28 giugno 2021.

of Naples. These papers confirm that Giovanni Novi played an important role in the Italian mathematical context of the mid-nineteenth century; however, inadequate attention was paid to him probably due to the fact that his unstable health conditions, that were critical since the early 1860s, followed by his premature death, greatly limited his scientific activity.

Giovanni Novi (1826-1866) was born in Naples. When he was 12 years old, he entered in the "Accademia militare della Nunziatella" and in 1844 he became member of the Corpo del Genio; because of his good mathematical background, he worked in the geodesy section of the Office of topography and took part in significant geodetic works in several Italian regions. Novi, with other officials, took part in the constitutional movements of 1848 and he was expelled from the Office. So, he left Naples and moved to Florence, where he formed close ties with Enrico Betti and with the Sicilian mathematician Placido Tardy (1816-1914), who had also moved to Florence in 1848, leaving his native Messina, where he had been professor of *Matematica sublime* since 1841. In Florence Novi taught mechanics and artillery, and later analysis and mechanics at the *Liceo Militare Arciduca Ferdinando*; in 1859 he left the position because he was appointed Chair of Algebra at the University of Pisa, succeeding Betti. In 1857 in Florence, Novi married Teresa Pozzolini and had two children. See [Palladino 2012]. Teresa was niece of Vincenzo Malenchini (1813-1881), political exponent of the Grand Duchy of Tuscany. She was daughter of Gesualda Malenchini (1809-1894) and sister of Cesira (1839-1914), wife of the philosopher Pietro Siciliani (1832-1885). See [Signorini-Visconti 2004].

Giovanni Novi, known primarily for his activity of translation and integration of important textbooks of mathematics, also wrote several original works: *Elementi d'aritmetica* [Novi 1857] as a preliminary text to the *Trattato d'aritmetica* [Novi 1862], translated from Bertrand's *Traité d'arithmétique* [Bertrand 1849]; *Memoria sopra le trasformazioni generali di date funzioni. Estratto di un opuscolo del sig. O. Schlömilch* [Novi 1850]; *Sul moto dei proietti nell'anima delle bocche da fuoco* [Novi 1854]. In 1856 he published the *Elogio di Carlo Gustavo Jacob Jacobi letto da Lejeune Dirichlet all'Accademia delle Scienze* [Novi 1856b]; in 1860 wrote a review of the three works: *Lezioni di Meccanica razionale di O. F. Mossotti, La Statica dei sistemi di forma invariabile di F. Brioschi, Milano 1859, Elementi di Meccanica razionale di D. Chelini delle Scuole Pie* [Novi 1860]. In 1864, Novi published his *Riduzione in serie delle facoltà analitiche* [Novi 1864a] and *Sugl'invarianti e i covarianti delle forme binarie* [Novi 1864b] in "Giornale di Matematiche" ([1]). We note that the "Giornale di Matematiche" – also known as the "Giornale di Battaglini" – had been published in Naples since 1863, and the subtitle expresses the aim of the project: "for the use of students in Italian Universities": it is one of the scientific journals conceived for re-launching of Italian mathematics. See [Enea 2017].

Last but not least, Novi is credited with the lithograph conserved in the library of the University of Pisa, without a title page, entitled *Lezioni di Artiglieria*, which carries a stamp classifying it as a donation from Betti.

The climate of Risorgimento fervour that pervaded pre-Unification Italy in the mid-nineteenth century was felt with particular vehemence at the universities. Such a fervour involved mathematicians who worked tirelessly in scientific as well as political-institutional arenas of the Italian state to equip it with new cultural structures, new instruments and new rules, and who contributed to elevating the status of Italian mathematics to the level enjoyed beyond the Alps. Correlated to the relaunching of Italian mathematics was the problem of drafting textbooks and manuals for students aimed at raising the level of teaching in schools and universities to that of the other European countries.

This was an objective that involved many mathematicians of the Risorgimento generation who engaged in writing manuals, sometimes based on their own lessons, and translations of foreign textbooks. Initially mathematics textbooks were the translation and adaptation of foreign volumes, but this methodology was only the starting point towards writing new manuals that were suitable for the future Italian schools. This operation was especially

([1]) The journal was founded by Giuseppe Battaglini (1826-1894), Vincenzo Janni (1819-1891) and Nicola Trudi (1811-1884).

coordinated by Enrico Betti in Pisa with the collaboration of his close friend Giovanni Novi. Betti, along with Luigi Cremona, Francesco Brioschi and others, was one of the period's leading mathematicians. In particular, Francesco Brioschi (1834-1897) re-established the *Annali di Matematica* in 1857; ([2]) he was the founder (in 1862) and first rector of the *Istituto Tecnico Superiore* – now *Politecnico di Milano* –, he was General Secretary for Public Education and he held several other political posts. Enrico Betti (1823-1892), who had introduced Galois' Theory to Italy, became head of the *Scuola Normale* in Pisa in 1865, an institute that produced many high level researchers in mathematics. He was also a member of the Italian Parliament, and in 1884 he became a senator. Luigi Cremona (1830-1903) was appointed Chair of *Geometria superiore* at the University of Bologna, where he also taught descriptive geometry. He then moved to Milan in 1866, where he taught graphical statics at the Polytechnic and advanced geometry at the *Scuola Normale* (annexed to the Polytechnic itself to train Technical Institute teachers). In 1873 he accepted an invitation to transfer to Rome as head of the *Scuola degli Ingegneri*. In Parliament he was recognized as an authority when it came to issues of education; hence Cremona was able to exert an enormous influence over the organization of mathematics education in Italy. He became a senator in 1879, and became vice-president of the Senate, and, in 1898, for just one month, Minister for Education. ([3])

Giovanni Novi's contributions to the teaching

Although Giovanni Novi was born and studied in Naples, he worked as a member of the team in Pisa, contributing to the process of renovation of education. From their correspondence, the close friendship and cooperation between Novi and Betti clearly emerges. In 1858 Novi was invited to Placido Tardy's home for an important meeting: Brioschi, Tardy, Betti and Angelo Genocchi (1817-1889) got together to talk about the *Annali di Matematica pura ed applicata*. In the letters on the 20th and 22nd of February, and on March 4th, 1861, in [Mercurio-Palladino 2011] we learn that Novi was also consulted, as teacher at the University of Pisa, about the reorganization of higher education of Mathematics, in connection with education of engineers. ([4]) The issue arose when the Minister of education Terenzio Mamiani, at the beginning of 1861, decided to write new regulations for higher studies, as part of the initiatives to adapt the *Legge Casati* (the first education law implemented by the united Italy) to the new political scenario. See [Polenghi 1993].

The group of excellent mathematicians that had formed mainly at the universities of Pavia and Pisa was deeply involved in advancing scientific research. In this perspective, we include: the resumption of publication of the "Annali di Tortolini"; Brioschi's founding of the "Politecnico di Milano"; the study tour of Enrico Betti, Francesco Brioschi and Felice Casorati (1835-1890) in several cities of Germany and France; and the adaptation of textbooks coordinated by Betti. Such a tour marked an important moment in the process of internationalization and scientific exchange between Italy and the other European countries. Cfr. [Bottazzini 1994], [Bottazzini-Nastasi 2013].

The educational aims of the group of mathematicians were to: connect teaching with the recent scientific research; investigate the relations between pure mathematics and its applications; contribute to the unification of school in Italy, unifying content, methods, languages and textbooks. In this context, in 1856, Betti began the translation of Bertrand's *Algebra* with additions and notes [Betti 1856]. In the preface he announced his intention of publishing a course in algebra; in those years, Betti held at home twice a week, for four of his best students, lessons to present

([2]) The "Annali di scienze matematiche e fisiche" was the first Italian international scientific journal, also known as "Annali di Tortolini" because of its founder Barnaba Tortolini, who published it in Rome from 1850 to 1857. In 1857, the journal was restructured to include an editorial board composed of Betti, Brioschi and Angelo Genocchi.

([3]) For Brioschi, Betti and Cremona there are extensive bibliographies; see [Raponi 1972], [Virgopia 1967], [Bottazzini-Rossi 1984].

([4]) On the problem of the education of engineers in Toscana, see [Soldani 2010].

parts of algebra that he could not teach in the university course. From these lessons, he hoped to compose a monograph, considering Bertrand's Algebra text as a basic textbook [Cerroni-Martini 2010, letter on 24 June 1859]. In 1857 Betti became chair of Algebra at the University of Pisa, and Betti's enterprise was taken up and completed by Novi who in 1863 published the *Trattato di algebra superiore – Parte Prima, Analisi Algebrica* [Novi 1863].

In the preface of the *Trattato*, Novi thanks Betti and explains the intentions and aims he pursued in compiling the treatise (pp. V-VI):

> Thanks to the works of recent scholars in geometry, the algebra is growing through such and with such notable improvements, that ancient treatises are not enough for its effective teaching, and they are not suitable for its present conditions. From this, the need, generally perceived, of a new treatise that, exposing the modern theories, guides the young students to the comprehension of the great problems of knowledge of the great problems in science through a not too uncomfortable path. [...] We have two aims, that are, first to compose a work that can serve all the needs of higher education, and second to offer young people who, once they have completed university studies, feel the inclination to continue along the difficult path of science, a book that introduced to reading of the Memories of the great geometers of our age. ([5])

Betti and Novi had planned the work divided into three parts; Novi only published the first part which contained the following topics: notions on the theory of combinations, complex numbers, limits and continuity of functions, series and convergence of series, exponential and logarithmic series, circular and hyperbolic series, continuous fractions. They had thought of including theories of determinants, symmetric functions, linear transformations for homogeneous functions (in the second part), irrational algebraic functions and algebraic resolution of equations (in the third part). Seemingly, Novi was working on the second part of the textbook when he died in 1866 [Saltini 2016, letter written on the 8th of January 1867]. As can be inferred from the only letter that has been found and that was sent from Pisa on December 28, 1863, Novi had also discussed this with Genocchi [Mercurio-Palladino 2011, p.227]. The letter also mentions Betti's partecipation in the treatise that Novi was writing. ([6])

As early as 1853, as can be inferred from the Novi-Betti correspondence, Novi had begun to devote himself to the compilation and translation of treatises aimed at teaching mathematics in schools. The first one he published is *Trattato d'aritmetica di Giuseppe Bertrand; prima traduzione italiana con note ed aggiunte di Giovanni Novi* [Novi 1856a], translated from Bertrand's *Traité d'arithmétique* [Bertrand 1849]. In the introduction he wrote that his main original contributions were on the theory of square and cubic roots, on complex numbers and in using letters and no numbers to prepare young pupils to the study of Algebra.

In 1857 he published *Elementi d'aritmetica* [Novi 1857]; and in the "Avvertenza" (warning to the reader) he wrote that it was a preparatory manual to [Novi 1856a]. The volume consists of 150 pages about addition and subtraction, multiplication, division, divisors of numbers and prime numbers, fractions, decimal numbers, complex numbers, ratios and proportions, rules of three, problems, and the metric system.

In 1858 Novi published the *Trattato di Geometria Elementare* [Novi 1858], an Italian translation of the French textbook by Antoine Amiot *Leçons nouvelles de géométrie élémentaire* [Amiot 1850]. In this circumstance, Luigi Cremona wrote a review that be-

([5]) "L'algebra per opera dei geometri moderni si è venuta accrescendo di tali e sì notevoli perfezionamenti, che gli antichi trattati non più bastano al suo efficace insegnamento né più rispondono alle sue condizioni presenti. Da ciò il bisogno, generalmente avvertito, di un nuovo trattato che, esponendo le teorie moderne, guidasse la gioventù studiosa per via non troppo disagevole alla intelligenza dei grandi problemi della scienza. [...] Noi abbiamo mirato a due fini, cioè, per primo comporre un'opera che potesse servire a tutti i bisogni dell'insegnamento superiore, e per secondo, offrire ai giovani che, compiuti gli studi universitari, si sentissero indotti a continuare nell'aspro cammino della scienza, un libro d'introduzione alla lettura delle Memorie dei grandi geometri dell'età nostra".

([6]) In this letter we can also read about encounters between Novi and Bernhard Riemann (1826-1866) while Riemann was staying in Pisa.

came very well-known, entitled "Considerazioni di storia della geometria in occasione di un libro di geometria elementare pubblicato a Firenze" [Cremona 1860], published in *Il Politecnico,* founded in Milan in 1839 by Carlo Cattaneo (1801-1869). Here Cremona explained the intent of the authors Betti, Novi and others in publishing their treatises, declaring that the basic idea was to assimilate modern discoveries into classical geometry, and admitting the absolute necessity of transforming the old books as to foster young people's participation to the progress of the science. In his paper Cremona mentioned (p. 286):

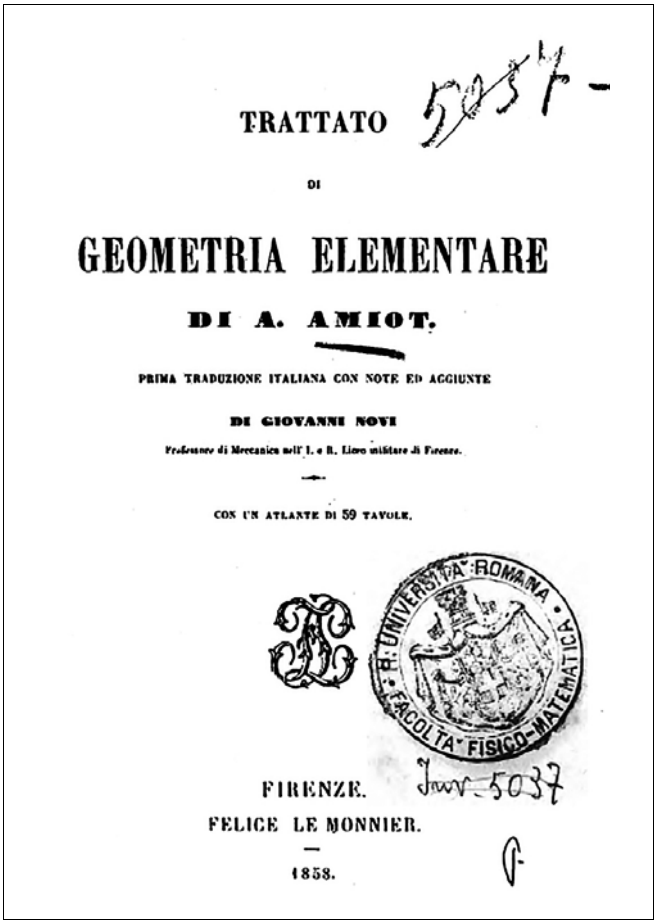
> [...] the *Trattato d'Aritmetica* by Giuseppe Bertrand, translated into Italian by professor Giovanni Novi [Novi 1856a]; appearing just two months later, the *Trattato d'Algebra Elementare* by Bertrand, translated by professor Enrico Betti [Betti 1856], and the *Trattato di Trigonometria* by Alfredo Serret, translated by professor Antonio Ferrucci [Ferrucci 1856]. A year after the same publisher brought out the *Elementi d'Aritmetica* [Novi 1857], written by professor Novi, because it served as an introduction to the *Trattato* by Bertrand. Now four months ago the *Trattato di Geometria Elementare* by A. Amiot [Novi 1858] appeared. ([7])

Cremona considered these as an organic *corpus* of textbooks aimed at the mathematical education of students, and regarding these books he wrote (p.323):

> The best books, indeed, the only truly good ones that a conscientious teacher of elementary mathematics could adopt for his teaching are the treatises of Bertrand, Amiot and Serret, so well translated and enlarged by those skilful Tuscans. ([8])

The *Trattato di Geometria Elementare*

In 1858 Novi published the *Trattato di Geometria Elementare,* the Italian translation of the textbook by Amiot *Leçons nouvelles de géométrie élémentaire*.


TRATTATO

DI

GEOMETRIA ELEMENTARE

DI A. AMIOT.

PRIMA TRADUZIONE ITALIANA CON NOTE ED AGGIUNTE

DI GIOVANNI NOVI

Professore di Meccanica nell' I. e R. Liceo militare di Firenze.

CON UN ATLANTE DI 59 TAVOLE.

R. UNIVERSITA' ROMANA FACOLTA' FISICO-MATEMATICA

FIRENZE.

FELICE LE MONNIER.

1858.

FIGURE 1 – Title page of Novi's treatise.

Antoine Alphonse Désiré Amiot was born in La Charité in 1812 and had begun his studies at the Collège in Bourges. He received a degree in mathematical sciences in 1837 and one in physical sciences in 1840. He taught "Mathématiques spéciales" ([9]) at the Collège Chaptal and at the Lycée Saint-Louis in Paris. He died in 1867. Amiot wrote

([7]) "Il *Trattato d'Aritmetica* di Giuseppe Bertrand, tradotto in italiano dal professore Giovanni Novi; scorsi appena due mesi tennero dietro il *Trattato d'Algebra Elementare* dello stesso Bertrand, tradotto dal professore Enrico Betti, e il *Trattato di Trigonometria* di Alfredo Serret, tradotto dal professore Antonio Ferrucci. Un anno dopo si pubblicavano dallo stesso editore [Le Monnier] gli *Elementi d'Aritmetica*, scritti dal professor Novi, perché servissero d'avviamento al *Trattato* del Bertrand. Ora da quattro mesi è uscito il *Trattato di Geometria Elementare* di A. Amiot".

([8]) "I migliori libri, anzi gli unici veramente buoni che un coscienzioso maestro di matematica elementare possa adottare nel suo insegnamento, sono i trattati di Bertrand, Amiot, Serret, così bene tradotti e ampliati da quei valenti toscani".

([9]) In France, "Mathématiques spéciales" was the name of an ancient chain of preparatory classes disappeared after the reform of higher education in 1997. They were preparatory courses with the main goal of training students for enrollment in the "Ecole normale supérieure", "Ecole polytechnique" and "Ecole centrale des arts et manufactures".

many textbooks for teaching mathematics in the schools, including *Leçons nouvelles de géométrie descriptive* (1853, composed in accordance with the renewed program for special mathematics), *Leçons nouvelles d'algèbre élémentaire* (1853), *Éléments de géométrie* (1860, composed in accordance with the new program for teaching the sciences in upper-level secondary schools, and followed by an addition to be used by students of "Mathématiques spéciales"), *Solutions raisonnées des problèmes* (1858, a kind of workbook with problems from the *Éléments de géométrie*, with some observations in the preface) and *Applications de la Géométrie élémentaire* (1856, written in accordance with the program for teaching the sciences in upper-level secondary schools). Reprints of the *Leçons nouvelles de géométrie élémentaire* were issued at least up until 1866 (Paris: Tandou & Cie). The French text written by Amiot is in large part a revision of Legendre's *Eléments de géométrie* [Legendre 1794], which was substantially a book of elementary geometry structured in a similar way to Euclid's *Elements*, but with simplifications of some of the proofs, using algebra and arithmetic to support the geometry. Giovanni Novi remarked that Legendre, "illustrious geometer, most meritorious in teaching", published the first edition of his work in 1794 and died in 1834, precisely in the period in which new theories were being worked out.

Amiot's book consists of eight chapters, four on plane geometry and four on solid geometry: *The straight line and the polygonal chain*; *Circumferences and circles*; *Proportional lines*; *Metric properties of figures*; *On plane and line*; *Polyhedra*; *Curved surfaces*; *Measurement of cylinders, cones and spheres – Regular polyhedra*. ([10]) In the Italian translation, the new contributions of Novi with respect to the French text are the ten notes (which are actually similar to chapters and consist of 131 pages in total) that Novi added at the end of the treatise. Furthermore, Novi added original contributions in the form of footnotes that enrich the volume as a whole and contain observations, explanatory notes or alternative proofs. Also, he added exercises at the end of chapters, linked to the theoretical additions that he inserted into the notes. Finally, Novi completed the treatise with original figures that did not appear in *Leçons nouvelles*.

Examining Novi's Italian version, we notice that the purpose of the less formal approach, based on reasoning, was to stimulate the pupils' imagination and develop their creativity. On the other hand, the approach to scientific research was still given by notions of projective geometry embedded into the text and into the final notes.

Thanks to his work, Novi's purposes were, on the one hand, to reduce the gap between students' knowledge and scientific research, strengthening the students' pre-university mathematical knowledge; on the other hand, he wanted to present mathematics in a more "creative" way, and so that it was less based merely on formal knowledge. In the introduction to the Italian version, Novi examined the didactical aims of the work and the reasons for the serious differences between manuals of elementary geometry and the researches in that field, tracing them back to the widely-held opinion that geometry was only of use if used for practical purposes; he wrote (p. VIII):

> [...] Translating this work and adding Notes to it, my intention is to inspire in students enthusiasm for the new, fruitful theories of the modern Geometry, and to raise the [level of] geometric studies in our schools. ([11])

Cremona wrote that Novi's translation was "destined almost exclusively to the development of recent theories that are only sketched out in the text" ("destinate quasi esclusivamente allo sviluppo delle teorie recenti soltanto abbozzate nel

([10]) *La Ligne droite et la Ligne brisée*; *De la Circonférence du Cercle*; *Des Lignes proportionnelles*; *Propriétés métriques des Figures*; *Du plan et de la Ligne droite*; *Des Polyèdres*; *Des Surfaces courbes*; *Mesure du Cylindre, du Cône et de la Sphère – Polyèdres réguliers.*

([11]) "Traducendo quest'opera e corredandola di Note altro non e' stato il nostro intendimento se non quello d'ispirare nei giovani l'amore a queste nuove, feconde e belle teorie di Geometria moderna, e di rialzare gli studi geometrici nelle nostre scuole".

testo") and that the brief notes inserted by the translator have "the aim of indicating new consequences of theorems set out by the author, or simpler proofs, or more general ways of considering certain arguments" ("allo scopo di indicare nuove conseguenze de' teoremi esposti dall'autore, o più semplici dimostrazioni, o maniere più generali di considerare certi argomenti") [Cremona 1860, p. 287].

Novi considered his ten notes to be a kind of "Complement to Geometry" and named them: *On the common measure of two lines; Polyhedra; On the method of projections; Anharmonic ratio; Involution; Homographic division; Centre of gravity. Centre of harmonic means. Poles and polars; Polar planes; Method of polar reciprocals; Conic sections.* ([12])

The first note contains a method formulated in Léger, *Mémoire sur les rapports et les restes* on incommensurability [Léger 1836] and is aimed at clarifying a few notions that are too concisely treated in the original; the second note is aimed at introducing the Poinsot's polyhedra of a higher order. The third note contains mentions of perspective, stereographic and orthographic projections. The four notes that follow complete the notions of modern geometry that are expounded in the text; here Novi picks up the projective properties developed in Chasles's *Géométrie supérieure* for figures located on both a plane and a sphere [Chasles 1852]. The eighth note contains the theory of poles and polars, on the plane and the sphere. The ninth note mentions the method of reciprocal polars. The final note concerns the conic sections and how modern methods are applied to these curves.

Novi added contributions on projective geometry mainly to books three and four of Amiot's textbook, dedicating to them ten footnotes and eight of the ten notes at the end of the book. Chapter III of Amiot's textbook begins with the definition of *harmonic proportion*; in the footnotes, the translator provides geometric definitions of *harmonic mean*, *centre of the harmonic mean* and *harmonic progression,* arriving at the geometric definition of *centre of gravity,* of which he then shows various properties. He finally comes to the generalised definition of harmonic mean and of centre of harmonic mean, and he invites the reader to consult [Chelini 1849] and [Poncelet 1826]. In the final note IV of the treatise, Novi picks up the projective properties developed in [Chasles 1852] and explains how to arrive at expressing the anharmonic ratio of four lines in function of the sines of the angles that they form reciprocally. In the second section he defines the *harmonic ratio of four points located on a circle*; in the third section Novi gives the definition of the *harmonic spherical bundle*. The fourth section is devoted to examples and applications of the theories described; here, Novi illustrates Desargues' theorem (and its converse). These theorems, he goes on to say, constitute the basis of what Poncelet had developed as the theory of homologous figures in [Poncelet 1822]. The second example he made is the well-known theorem, whose generalisation is due to Blaise Pascal, known as the *mysterium hexagrammaticum* [Pascal 1640]. Novi's fifth note is dedicated to involution. Here Chasles' definitions and properties of *double points* and *point central of involution* are given. After having also stated another important theorem of Desargues on complete quadrilaterals, Novi applies the theory of involution to the solution of some problems. The note concludes with the definition of six points in *spherical involution.*

Chapter V of the third book of Amiot's textbook deals with similitudes. In a note Novi inserts a proof as alternate to two problems proposed by Amiot on the construction of the fourth point of a harmonic proposition and of the fourth line of a harmonic bundle, using a property of the complete quadrilaterals explained earlier. Novi's sixth note is dedicated to *homographic division*. In the first part he gives the basic definition and explains how to construct two homographic divisions. Then, he provides a summary of what Chasles elaborated in chapter VII of [Chasles 1852]. The eighth and ninth notes develop the fourth book of Amiot's treatise; they concern the metric properties of the figures,

([12]) *Sulla comune misura di due rette*; *Poliedri*; *Sul metodo delle proiezioni*; *Rapporto anarmonico*; *Involuzione*; *Divisione omografica*; *Centro di gravità. Centro delle medie armoniche. Poli e polari; Piani polari*; *Metodo delle polari reciproche*; *Sezioni coniche.*

and they are devoted to the notions of *pole*, *polar* and *polar planes*, and to the method of reciprocal polars formulated in [Poncelet 1822]. Then, Novi shows that two reciprocal polar figures are such that each is the locus of the poles of the lines tangent to the other, and each is simultaneously the envelop of the polar lines of the points of the other; it is understood that these polars and poles are always taken with respect to the conic directrix. By means of these methods, from any theorem of geometry that involves only projective properties, another can be derived that is its polar reciprocal. However, if the proposed theorem contains metric properties or angle ratios, then it is possible to derive many others, each of which corresponds to a special conic directrix. As an example of this assumption, Novi introduces Pascal's mystic hexagram theorem and the theorem thus derived thanks to [Brianchon 1806], that are mutually polar. Novi then shows how some of the other theorems he had presented in the preceding notes are also reciprocally polar.

The third note is dedicated to the method of projection and to stereographic projections. Novi gives the definition of *centre of projection* and *surface of projection,* and then he goes on to demonstrate various problems and propositions that follow from them. This is followed by the proofs of the fundamental principles of stereographic projection.

In the tenth note, after having begun with a few fundamental notions, Novi goes on to examine how to apply the method of polar reciprocals and the method of projections to the conic sections, immediately illustrating the theorem on which the former is based. Using Pascal's theorem Novi solves the problem of how to construct a conic that passes through five given points.

Another feature in Novi's work is that he chose to use some of the exercises that Amiot proposed within the text to illustrate these new theories, spurred by the important role that some of these problems played in further developments of the sciences, and in hope of introducing the young students to the new and fruitful geometric theories, with the aim of improving geometry education in schools; he also argued that a comparison of various methods was useful and effective in promoting modern studies. Surely drawing on his own experience as a secondary school professor, he used this method several times in his translation; one such example is the theorem of Pascal's mystic hexagon.

We can see that in the Italian translations, in general, the text appears much more accurate and precise, and rich in detailed historical references, which are also interesting from a didactic point of view in that they provide on a perspective on the Italian author's intention of making geometry more enjoyable for students; some of his historical references are starting points that lead to the idea of a "creative mathematics".

Indeed, such position also emerges in the second book of the *Leçons nouvelles de géométrie*, concerning circumferences and circles. Novi's original interventions are mostly about star polygons. Amiot had left these out, referring the reader to Poinsot's "Mémoire sur les polygones et les polyedres" [Poinsot 1809]; Novi first recalls that star polygons are classified by order and kind, and then he inserts, in the first note to the book (p. 64), the sort of geometric definition of prime numbers given by Poinsot: if n is the order and h is the species of a star polygon, it can happen that uniting n points A, B, C, ... of h in h one passes by all of these points before returning to the point of departure; the number h will then necessarily be prime with respect to n. Further, if uniting in the same way several points at intervals that are almost equal, one never returns to the first one without passing through all the others, then it can be said that n is prime. Starting from these definitions, in the second note (p. 66), Novi proposes various ideas for calculating the sum of the interior angles in the previously defined star polygons and for searching for polygons (star or usual) in which the sum of the interior angles is the same.

The subject is useful to tackle problems and reflections on similar, equiangular, equivalent polygons and, as Cremona himself underlined in his review, on isoperimetric polygons. This is an example of what Novi wanted to do: to present mathematics, when possible, with a less formal and more creative approach. It seems interesting to compare Novi's presentation of this argument with the way it is presented in one of the most popular books in the Lombard school, such as the

text *Geometria piana e solida* written by Francesco Toffoli ([13]) [Toffoli 1846]. As for isoperimetric problems, in Toffoli's superficial presentation and formal exposition there is no opportunity for personal reasoning; this happens more in problems placed at the end of chapters. An example is the following: "What are the measures in meters of the side of the equilateral triangle and the side of the regular hexagon which are isoperimetric to a circle having a radius of meters 3.40; and what is the area in square meters of these three geometric figures?" ([14]) [Toffoli 1846, p. 95].

The author then showed all the calculations needed to solve this and similar problems:

As we can see, the solution shown above is full of calculations and formulas, numbers and mechanical procedures; there are no clarifying pictures, either. While Novi attempted to start from objects that intrigue students, in Toffoli we see calculations and formalisms. The problems added by Novi to Amiot's work were the results of his personal research (also from a historical point of view) on subjects, which suggest a more appealing and less notional approach to learning. Cremona himself wrote about the Toffoli's text in the final note of his review [Cremona 1860, p. 322]:

> Today the foreign yoke is no longer on our backs to impose on us the loathsome textbooks of Močnik ([15]), Toffoli, etc., which for many years have inundated our schools, and would have completely barbarised them if all the teachers had been docile and served the interest of the Gerold company – it would also be now be time to throw into the fire certain terrible books of mathematics that are still being used in some of our lyceums and which comprise a fiery accusation against those who adopted them. ([16])

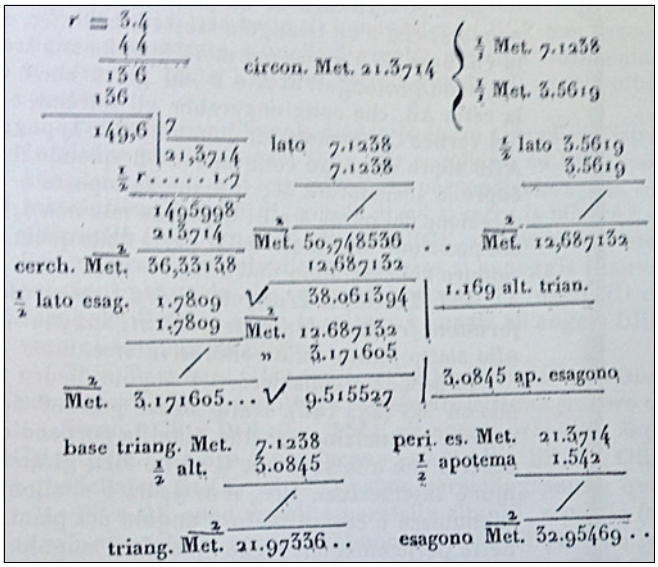


FIGURE 2 – Figure from [Toffoli 1846, p. 96].

Also, in the part relating to solid geometry, Novi chose to insert the theory on star polyhedra starting from [Cauchy 1813a], [Cauchy 1813b], [Poinsot 1809], [Poinsot 1858]. He shows that the extension of the faces or the edges of the five regular polyhedra of the first species cannot produce other regular polyhedra except for those due to Poinsot. He gives instructions for their constructions; indeed, the book concludes with five theorems added by the translator aimed at proving (and giving instructions for the construction) that, given one side, it is always possible to create a tetrahedron, a hexahedron, an octahedron, a dodecahedron and a regular icosahedron.

The topics selected by Novi, star polygons, star polyhedra but also transformations, linked the teaching with research or made mathematics an instrument for developing not only rational faculties but also those related to creativity and imagination.

([13]) Francesco Toffoli, professor of mathematics and physics, author of several books of arithmetic and algebra for teaching in secondary schools, printed in Vienna by the Imperial Administration.

([14]) "Di quanti metri sono il lato del triangolo equilatero e dell'esagono regolare, isoperimetrici al cerchio di raggio 3.40 metri; e di quanti metri quadrati è l'area di ciascuna di queste tre figure?".

([15]) Franz Ritter von Močnik (1814-1892), author of books on geometry and algebra for teaching in secondary schools.

([16]) "Ora che il giogo straniero non ci sta più sul collo a imporci gli scelleratissimi testi di Moznik, Toffoli, ecc., che per più anni hanno inondate le nostre scuole, e le avrebbero del tutto imbarbarite se tutt'i maestri fossero stati docili a servire gl'interessi della ditta Gerold – ora sarebbe ormai tempo di gettare al fuoco anche certi libracci di matematica che tuttora si adoperano in qualche nostro liceo e che fanno un terribile atto d'accusa contro chi li ha adottati".

Conclusions

Before the unification of Italy and up to 1867, the year in which Minister of Education Michele Coppino introduced the use of Euclid's *Elements* for secondary school teaching, most schools in the various Italian states had adopted translations of foreign works. The one who inspired the mathematics programs and related instructions regarding methodologies was Cremona, who, fully realizing what was said in his review, restored Euclid as a textbook for lower and upper secondary schools. In 1867 the commission nominated by the Italian government to oversee the mathematics program for the schools of the newly formed Kingdom of Italy, of which Cremona himself was a member, stipulated that geometry was to be taught using the Euclidean method, and in particular that Euclid's *Elements* were to be taught in lower-level (*ginnasio*) and upper-level (*liceo*) secondary schools. Specifically, the program called for the study of Euclid's I, II, III, IV, V, VI, XI.

The introduction of the *Elements* led to the disappearance of a large part of the foreign texts. Further, collaborating with Brioschi on the Italian translation of the *Elements* published in 1867 [Betti-Brioschi 1867], Cremona proposed "banning innumerable worthless book, compiled for pure speculation, that infest those very schools where there is an even greater need for scientific rigour and soundness of method in textbooks" ("sbandire innumerevoli libercoli, compilati per pura speculazione, che infestavano appunto quelle scuole dove è maggiore pei libri di testo il bisogno del rigore scientifico e della bontà del metodo") and to favour the publication of good Italian manuals. ([17])

In the period immediately preceding the Italian Unification, Novi's text can be considered as a transitional enterprise in the process of renovation of teaching and school. In the introduction to the Italian version, we read that it was the publisher Le Monnier who invited Novi to propose a treatise on geometry that could be used in schools, and that Novi had chosen Amiot. In order to explain his choice, in reference to Amiot's work, Novi writes that the new methods lend themselves with ease and generality to solving geometric problems, and that they are, therefore, useful for applications of science, like, according to Novi, what Charles Julien Brianchon had done with the *Application de la Thèorie des Transversales* [Brianchon 1818] or what Jacob Steiner had done with *Die geometrischen Konstructionen* [Steiner 1833], showing how these concepts could be quite valuable to military men, engineers and land surveyors. Novi's work of integrating new notions into Amiot's textbook turns out to be particularly interesting: a useful starting point for disseminating the new theories. Even though his book did not yet fully conform to the requirements that were being defined, it was in any case an attempt that was almost unique at that time.

The return to the Euclide's *Elements* could be seen as step back in this methodology of the teaching of mathematics, but we have to take into consideration the fact that the Italian schools should be provided with valid texts. The authors themselves of the new Euclidean text wrote that mathematics must not be considered as a set of concepts useful for practical everyday needs, but mainly as an intellectual and cultural medium, as a means for training the mind. Concerning this perspective, a letter of Cremona to Betti on the 8th of September, 1869, states [Gatto 1996, pp. 52-53]:

> I am sure that modern methods, especially of Steiner and Staudt, are intended to renew all geometric knowledge, from the foundations; thanks to those methods, even the most basic things can be treated in a simpler, more original, more fruitful way. But such methods cannot be introduced into schools until an elementary book, written with this intent, exists and such a book does not exist, and I don't think that for now there is anyone who wants to or can do it. Until the distant day when this radical reform can be implemented, I think that the *Euclide* will always remain the best text for teaching geometry in classical schools. ([18])

([17]) Francesco Brioschi, Luigi Cremona, "Al signor Direttore del Giornale di matematiche ad uso degli studenti delle Università italiane", *Giornale di Matematiche*, 7, 1869, Napoli, pp. 51-54.

([18]) "Io sono convinto che i metodi moderni, specialmente di Steiner e Staudt sono destinati a rinnovare tutto lo scibile geometrico, sin dagli elementi; con quei metodi, anche le cose più elementari possono essere trattate in modo più semplice,

The letter was written in the context of the polemic which involved not only Italian mathematicians about the use of *Euclid's Elements* as a textbook for teaching geometry. For the discussion see [Furinghetti-Somaglia 2005], [Giacardi 2006], [Menghini 2006].

In 1871, the Italian Ministry of Agriculture, Industry and Commerce approved a school reform on the teaching of geometry in Technical Institutes, and Cremona was member of the commission that introduced the teaching of the fundamental principles of projective geometry into Technical Institutes. Cremona's *Elementi di Geometria Proiettiva*, published in 1873, was written to satisfy the requirements proposed in the reformed program. This was supposed to have been a textbook used to introduce the methods of modern geometry into schools, and it was aimed at technical institutes where future engineers were educated, although this does not mean that it was written in an elementary way didactically speaking, it was directly usable by students without the didactic mediation of the professor; [Di Sieno 2006], [Menghini 2006], [Pepe 2013]. In the preface, Cremona declares the aim of disseminating in Italian schools the theories whose foundations were found in Euclid, Apollonius, Chasles and von Staudt, stating their asset of being easily understood and requiring few preliminary notions.

What Cremona wrote in the introduction to his text on the projective geometry is not very different from what we read in the *Elements*. He argued that the role of mathematics in the educational process is the same, both at the Technical Institutes and at the Lyceums, and for the education of the future lawyers and engineers. According to Cremona, this very strong *formative* value of geometry should be considered the fundamental subject of a new didactic policy, where scientific education becomes the main focus of the training for the new ruling class. The Technical Institute, in this perspective, gains almost the same importance of classical studies at the lyceum and engineers become essential components of the ruling class of the new Italy.

It is possible that Cremona's manual was intended not only to raise the level of scientific culture in technical institutes, but also to constitute an experiment to promote the use of methods of projective geometry in secondary schools.

It is specified that in 1870 the Italian Ministry of education reduced the integral use of Euclid's *Elements* leaving teachers free to choose a treatise on solid geometry; later this obligation was eased and only survived in observance of the method but not in the use of the Euclidean text. ([19])

About the aims that Novi, Betti and Cremona had in mind, when Betti and Novi decided to translate and integrate foreign texts, and when Cremona gave his full approval in his review, we can identify two different but connected aspects. On the one hand, a new method of teaching mathematics, and in particular geometry, was necessary; on the other hand, the new textbooks reveal the personal pedagogical style of the mathematical team. Their methods make possible a more modern teaching of geometry, that is better designed to teach specific abilities to future engineers and, in general, to the future national leadership class. This is, indeed, an opening to projective geometry (according to the vision that shortly afterwards will become dominant in the European polytechnics, that is to see projective geometry as the springboard towards graphical statics) and at the same time a way of seeing geometry as an intellectual training ground for the development of students' rational capacities. The spirit of the new math is, in the end, even that of current problems: a math oriented towards education and towards practical aspects. Novi's text represented a fusion between the practical and theoretical needs and an integration between applied and educational aspects in the teaching of mathematics. Furthermore, Novi reveals a specific

più originale, più fecondo. Ma tali metodi non si potranno introdurre nelle scuole sinché non esista un libro elementare, scritto appositamente: e un tal libro non esiste, e non credo che per ora ci sia chi voglia o possa farlo. Fino al giorno ancor lontano in cui tale riforma radicale potrà essere attuata, credo che l'Euclide rimarrà sempre la miglior guida per l'insegnamento della geometria nelle Scuole classiche".

([19]) Cfr. "Modificazioni ai programmi stabiliti per i ginnasi e per i licei col regio decreto del 10 ottobre 1867", circular n.287 of the Italian "Ministero della pubblica istruzione", November the 1st, 1870.

taste for mathematics that relegates rote calculation to the background, focusing instead on reasoning and visualization, and on the extensive use of geometric transformations rather than on memorization of formulas.

Novi's translation of the treatise on geometry into two separate parts of plane geometry and solid geometry enjoyed various new editions over time, at least until 1918. The following passage added by Novi himself to the preface to the 1861 edition of his translation (p. IX) is an indication of how widely distributed the text *Trattato di Geometria elementare* was:

> The hopes that guided us, when we offered to young students this translation of ours, were not disappointed. Amiot's Geometry has been adopted in many Italian schools, and it has been received favourably by authoritative judges. And the desire we expressed two years ago, that the teaching of geometry would be amplified, has already started to be put into action by the creation of chairs for higher geometry in two of the most important Italian universities, those of Bologna and Naples, which have been given to professors Cremona and Battaglini, whose valuable work influenced the government's choice. We are pleased to have contributed, though very modestly, to this rebirth of geometric studies in our nation. ([20])

We can conclude that Giovanni Novi belonged to the "Risorgimento" generation of Italian mathematicians who performed a genuine scientific and organisational miracle, carrying Italian mathematics from a dignified by marginal position on the world's scene to the level of excellence. They worked with the intention of creating new cultural institutions, reforming those already existing, founding specialised journals, giving new impetus to mathematical research in order to raise it to the levels of other European countries; they were also strongly influential in the reform of schools and universities. Novi's attempt was a rather successful one, in a phase of transition that paved the way for the involvement of great mathematicians in writing books that were educationally more effective and closer to the scientific results that were being achieved at that very moment. We need only cite the book on projective geometry by Cremona, those on elementary geometry by Veronese, D'Ovidio and others, and the books by Enriques and Amaldi, destined to be reprinted for an entire century.

([20]) "Le speranze che ci guidarono, quando offrimmo alla gioventù studiosa questa nostra traduzione, non sono state deluse. La Geometria di Amiot è stata adottata in monte scuole italiane, ed accolta con favore da giudici autorevoli. E il desiderio da noi manifestato or sono due anni, che venisse ampliato l'insegnamento della geometria, ha già ricevuto un principio di attuazione nella creazione di cattedre di Geometria superiore in due fra le principali Università italiane, la Bolognese e la Napoletana, alle quali sono state chiamate i professori Cremona e Battaglini, che pregevoli lavori indicavano alla scelta del Governo. Noi siamo lieti di aver contribuito, comunque assai modestamente, a questo rinascimento degli studii geometrici nel nostro paese".

References

[Amiot 1850] Amiot A. *Leçons nouvelles de géométrie élémentaire*, Paris: Dezobry et E. Magdeleine.

[Bertrand 1849] Bertrand J. *Traité d'arithmétique*, Paris: Hachette.

[Betti 1856] Betti E. *Trattato di algebra elementare di J. Bertrand. Prima traduzione italiana con note ed aggiunte di E. Betti*, Firenze: Le Monnier.

[Betti-Brioschi 1867] Betti E., Brioschi F. G*li Elementi d'Euclide con note aggiunte ed esercizi ad uso de' ginnasi e de' licei*, Firenze: Le Monnier; Libri I-III 1867, IV-VI 1868, XI-XII e Appendice 1868.

[Bottazzini-Rossi 1984] Bottazzini U. Rossi L. "*Cremona Luigi*", Dizionario Biografico degli Italiani, Volume 30, *ad vocem*.

[Bottazzini 1994] Bottazzini U. *Va' pensiero. Immagini della Matematica nell'Italia dell'Ottocento*, Bologna: Il Mulino.

[Bottazzini-Nastasi 2013] Bottazzini U., Nastasi P. *La Patria ci vuole eroi. Matematici e vita politica nell'Italia del Risorgimento*, Zanichelli editore.

[Brianchon 1818] Brianchon C.J. *Application de la thèorie des transversales*, Paris: Bachelier.

[Cauchy 1813a] Cauchy A.L. "*Recherches sur les polyèdres: premier mémoire*", Journal de l'École polytechnique, XVIe cahier, tome IX, pp.7-25.

[Cauchy 1813b] Cauchy A.L. "*Sur les polygones et les polyèdres: second mémoire*", Journal de l'École polytechnique, XVIe cahier, tome IX, pp. 26-38.

[Cerroni-Martini 2010] Cerroni C., Martini L. *Il carteggio Betty-Tardy (1850-1891)*, Milano: Mimesis.

[Chasles 1852] Chasles M. *Traité de géométrie supérieure*, Paris: Bachelier.

[Chelini 1849] Chelini D. *Sui centri dei sistemi geometrici*, tipogr. Marini e Morini, vol. V.

[Cremona 1860] Cremona L. "*Considerazioni di storia della geometria, in occasione di un libro di geometria elementare pubblicato recentemente a Firenze*", Il Politecnico, Milano: Editori del Politecnico, vol. IX, pp. 286-322.

[Di Sieno 2006] Di Sieno S. "*Luigi Cremona e la formazione tecnica pre-universitaria nella seconda metà dell'Ottocento*" in Giacardi L. (Ed.) *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, Lugano: Agorà Publishing, pp. 99-124.

[Enea 2017] Enea M.R. "*Il Giornale di Matematiche di Battaglini*", Cultura e Società. Rivista dell'Unione Matematica Italiana, Serie 1, Vol. 2, n.1, p. 63-80.

[Ferrucci 1856] Ferrucci A. *Trattato di trigonometria di J.A. Serret; prima traduzione italiana con note e ed aggiunte di A. Ferrucci*, Firenze: Le Monnier.

[Furinghetti-Somaglia 2005] Furinghetti F., Somaglia, A. "*Emergenza della didattica della matematica nei primi giornali matematici italiani"* in Moreira D., Matos J.M (Eds.) *História do ensino da Matemática em Portugal*, Beja: Sociedade Portuguesa de Ciências da Educaçao, pp. 59-78.

[Gatto 1996] Gatto R. "*Lettere di Luigi Cremona a Enrico Betti (1860-1890)*" in Menghini M. (Ed.) *Per l'Archivio della Corrispondenza dei Matematici Italiani. La corrispondenza di Luigi Cremona (1830-1903)*, vol. III, Quaderni P.RI.S-T.EM. Università "L. Bocconi", Milano, n. 9, pp. 7-90.

[Giacardi 2006] Giacardi L. "*L'insegnamento della Matematica in Italia dall'Unità all'avvento del fascismo*" in Giacardi L. (Ed.) *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, Lugano: Agorà Publishing, pp. 1-63.

[Legendre 1794] Legendre A.M. *Éléments de Géométrie*, Paris: Imprimérie Firmin Didot.

[Léger 1836] Léger È. "*Mémoire sur les rapports et les restes des quantités incommensurables*", Journ. Math., pp. 93-101.

[Luceri 2013] Luceri F. "*Il carteggio familiare di Pietro Siciliani (1850-1914)*", Lecce: Il Grifo.

[Luceri 2015] Luceri F. "*Pietro Siciliani e Cesira Pozzolini. Filosofia e letteratura*", Lecce: Il Grifo.

[Menghini 2006] Menghini M. "*The role of Projective Geometry in Italian education and institutions at the end of the 19th century*", International Journal for the History of Mathematics Education, vol. 1, pp. 35-55.

[Novi 1850] Novi G. "*Memoria sopra le trasformazioni generali di date funzioni. Estratto di un opuscolo del sig. O. Schlömilch*", Annali di Scienze matematiche e fisiche, Roma, t. I, pp. 517-527.

[Novi 1854] Novi G. "*Sul moto dei proietti nell'anima delle bocche da fuoco*", Annali di Scienze matematiche e fisiche, Tipografia delle belle arti, Roma, pp. 365-408.

[Novi 1856a] Novi G. *Trattato d'aritmetica. Prima traduzione italiana con note ed aggiunte di Giovanni Novi*, Firenze: Le Monnier.

[Novi 1856b] Novi G. "*Elogio di Carlo Gustavo Jacob Jacobi letto nell'Accademia delle Scienze di Berlino il 1º di luglio 1852 da Lejeune Dirichlet*", Annali di Scienze matematiche e fisiche, t. VII, Tipografia delle belle arti, Roma, pp. 342-372.

[Novi 1857] Novi G. *Elementi d'aritmetica*, Firenze: Le Monnier.

[Novi 1858] Novi G. *Trattato di Geometria elementare di A. Amiot, prima traduzione italiana con note ed aggiunte di Giovanni Novi*, Firenze: Le Monnier.

[Novi 1860] Novi G. "*Lezioni di Meccanica razionale di O. F. Mossotti, La Statica dei sistemi di forma invariabile di F. Brioschi, Milano 1859, Elementi di Meccanica razionale di D. Chelini delle Scuole Pie Bologna*", Annali di Matematica pura e applicata, Roma, t. 3, pp. 245-251.

[Novi 1863] Novi G. Trattato di Algebra superiore, Parte I. Analisi algebrica, Firenze: Le Monnier.

[Novi 1864a] Novi G. "*Riduzione in serie delle facoltà analitiche*", Giornale di Matematiche, vol. II, Benedetto Pellerano Editore, Napoli, pp. 1-46.

[Novi 1864b] Novi G. "*Sugl'invarianti e i covarianti delle forme binarie*", Giornale di Matematiche, vol. II, Benedetto Pellerano Editore, Napoli, pp. 306-330.

[Mercurio-Palladino 2011] Mercurio A.M., Palladino N. "*La corrispondenza Giovanni Novi-Enrico Betti*", Rendiconto dell'Accademia delle Scienze Fisiche e Matematiche di Napoli, serie IV, vol. 78, pp. 223-292.

[Palladino 2012] Palladino N. "*Giovanni Novi (1826-1866). La corrispondenza con Enrico Betti ed il suo contributo matematico*", in Pepe L. (Ed.) Europa Matematica e Risorgimento Italiano, Bologna: Clueb, pp. 189-198.

[Pascal 1640] Pascal B. *Essay pour les coniques*, Paris.

[Pepe 2013] Pepe L. "*Insegnamenti matematici nell'Italia unita*" in Lacaita C.G., Fugazza M. (Eds.) *L'istruzione secondaria nell'Italia unita. 1861-1901*, Milano: FrancoAngeli, pp. 147-156.

[Poinsot 1809] Poinsot L. "*Mémoire sur les polygones et les polyédres*", Journal de l'École polytechnique, Paris, Imprimerie Imperiale, pp. 16-48.

[Poinsot 1858] Poinsot L. "*Note sur la théorie des polyédres*", Comptes rendus des Séance de l'Académie des Sciences, tome quarante-sixième.

[Polenghi 1993] Polenghi S. *La politica universitaria italiana nell'età della Destra storica*, Brescia: Editrice La Scuola.

[Poncelet 1822] Poncelet J.V. *Traité des propriétés projectives des figures*. Paris: Bachelier.

[Poncelet 1826] Poncelet J.V. "*Mémoire sur les centres des moyennes harmoniques*", Journal für die reine und angewandte Mathematik, Berlin, pp. 213-272.

[Pozzolini 1875] Pozzolini C. "*Lettere e frammenti inediti di Antonietta Pozzolini*", Letture di Famiglia, Tip. di M. Cellini e C.

[Pozzolini 1904] Pozzolini C. *Lettere da Parigi*, Firenze: Tipografia Barbera.

[Raponi 1972] Raponi N. "*Brioschi Francesco*", Dizionario Biografico degli Italiani, Volume 14, *ad vocem*.

[Saltini 2016] Testi Saltini P. *Le lettere di Enrico Betti (dal 1860 al 1886) conservate presso l'Istituto Mazziniano di Genova*, Milano.

[Signorini-Visconti 2004] Signorini M.A., Visconti A. "*Il salotto di Gesualda e Cesira Pozzolini nella Firenze del 1859*" in Betri M.L., Brambilla E. (Eds.) *Salotti e ruolo femminile in Italia*, Venezia: Marsilio, pp. 381-403.

[Soldani 2010] Soldani S. "*Ingegneri e studi di ingegneria nella Firenze di metà Ottocento*", in Angotti F., Pelosi G., Soldani S. (Eds.) *Alle radici della moderna ingegneria. Competenze e opportunità nella Firenze dell'Ottocento*, Firenze: University Press, pp. 1-28.

[Steiner 1833] Steiner J. *Die geometrischen Konstructionen*, Berlin: Ferdinand Dümmles.

[Toffoli 1846] Toffoli F. *Geometria piana e solida*, Venezia: A. Santini e figlio, Tip. Libr. Editori.

[Virgopia 1967] Virgopia N. "*Betti Enrico*", Dizionario Biografico degli Italiani, Volume 9, *ad vocem*.

Nicla Palladino

Nicla Palladino è professore presso l'Università degli Studi di Perugia per il settore MAT/04. Laureata a Napoli alla ''Federico II'', è Dottore di ricerca in Matematica applicata e Informatica. A partire dagli anni del dottorato, ha rivolto i suoi studi principalmente a strumenti e modelli di superfici matematiche costruiti nell'800, all'esame delle corrispondenze epistolari tra matematici durante il periodo risorgimentale e a varie questioni di carattere analitico ad esse connesse. In questi ultimi anni si è dedicata all'approfondimento di aspetti storici legati a problemi di Geometria elementare e a relative applicazioni al campo della didattica.